For all questions, answer E. "NOTA" means none of the above answers is correct.

1. At what point is the tangent line to the graph of $y=\arctan(x)$ parallel to the tangent line to the graph of $y=\ln(x)$ at the point (1,0)?

A) $(0,0)$ B) $\left(1,\frac{\pi}{4}\right)$ C) $\left(-1,-\frac{\pi}{4}\right)$ D) $\left(\frac{\sqrt{3}}{2},\frac{\pi}{3}\right)$ E) NOTA

2. If $\int_2^9 \frac{e^{4x}}{e^{4x-1}}\,dx = Ae^B$ where A and B are integers, then find $A+B$.

A) 6 B) 7 C) 8 D) 9 E) NOTA

3. Find the range of the function $f(x)=\frac{x+1}{x^2+1}$.

A) $\left[\frac{1-\sqrt{2}}{4},\frac{1+\sqrt{2}}{4}\right]$ B) $\left[\frac{1-\sqrt{2}}{3},\frac{1+\sqrt{2}}{3}\right]$ C) $\left[\frac{1-\sqrt{2}}{2},\frac{1+\sqrt{2}}{2}\right]$

D) $\left[1-\sqrt{2},1+\sqrt{2}\right]$ E) NOTA

4. Let $F(x)=5(x+1)^6$, $G(4)=2$ and $G'(4)=\frac{1}{9}$. Find $R'(4)$ if $R(x)=F(G(x))$.

A) 162 B) $\frac{320}{3}$ C) $\frac{31250}{3}$ D) 810 E) NOTA

5. The edge of a cube is increasing at a constant rate of 196 centimeters per hour. Find the ratio of the rate at which the volume of the cube is increasing to the rate at which the surface area of the cube is increasing when the surface area of the cube is 216 square centimeters.

A) 1:4 B) 3:4 C) 3:2 D) 4:1 E) NOTA

6. Use differentials to approximate the volume (in cubic inches) of a spherical shell whose inner radius is 3 feet and whose thickness is $\frac{1}{16}$ inch.

A) 2.25π B) 18π C) 144π D) 288π E) NOTA

7. Given that $h(x)=\dfrac{f(x)}{x^2-8x+16}$ and that $\lim\limits_{x\to 4} f(x)=0$, then which one of the following statements is always true of $h(x)$?

A) $\lim\limits_{x\to 4} h(x)=0$
B) $\lim\limits_{x\to 4} h(x)$ does not exist.
C) $\lim\limits_{x\to 4} h(x)$ exists but may equal something other than zero.
D) $\lim\limits_{x\to 4} h(x)$ may or may not exist.
E) NOTA

8. Solve the differential equation $\dfrac{dy}{dx}=\dfrac{\sqrt{(1+y^2)^5}}{6y}$ subject to the initial condition $y=\sqrt{3}$ when $x=-1$.

A) $4x+6=16(1+y^2)^{-1.5}$
B) $4x+3=8(1+y^2)^{-3/2}$
C) $x=-2(1+y^2)^{-1.5}-.75$
D) $-\dfrac{1}{3}(1+y^2)^{-3/2}=\dfrac{1}{6}x$
E) NOTA

9. The area bounded by the graphs of $f(x)=x$ and $g(x)=x^2-x$ is divided into two regions of equal area by the line $x=k$. Find the area of the region bounded by $g(x)$ and the x-axis from $x=2k$ to $x=3k$.

A) $\dfrac{23}{6}$ B) $\dfrac{25}{6}$ C) $\dfrac{29}{6}$ D) $\dfrac{31}{6}$ E) NOTA

10. For $f(x)=3\sin x+\cos x-e^{-x}+\ln x^3+4x+7$, find $f'''(1)+f''(1)+f'(1)+f(1)$.

A) 14 B) 15 C) 17 D) 21 E) NOTA

11. The position of a particle moving along a number line is given by $s(t)=t^5-2t^4+3t^3-6t^2+5t-8$ where t is measured in seconds and $t\geq 0$. Find the average rate of change of the derivative of the acceleration function during the first 5 seconds of the particle's motion.

A) 398 B) 327 C) 252 D) 179 E) NOTA

12. If a function $g(x)$ has a continuous derivative $h(x)=\frac{d}{dx}g(x)$ for $-\infty<x<\infty$, then $\int_a^b h(2x)dx=$

A) $\frac{g(2b)-g(2a)}{2}$ B) $g(2b)-g(2a)$ C) $2[g(2b)-g(2a)]$ D) $2\left[g\left(\frac{b}{2}\right)-g\left(\frac{a}{2}\right)\right]$ E) NOTA

13. If $y=\ln|x|-2x^3-5x^2+3$, then find the derivative of y with respect to $\frac{1}{x^2+2}$ at the point where $x=-2$.

A) $-\frac{7}{2}$ B) $-\frac{9}{2}$ C) $\frac{63}{2}$ D) $\frac{81}{2}$ E) NOTA

14. Find the equation of the line normal to the graph of $y=x^6+x^4-2x^2-3x+1$ at the point where $x=1$.

A) $x+3y+5=0$ B) $x+3y-5=0$ C) $3x-y-5=0$ D) $3x-y+5=0$ E) NOTA

15. A baseball is dropped from the top of a cliff 144 *ft* above the Colorado River. One second later, a second baseball is thrown from the same location on the same cliff. With what velocity (in *ft*/sec) must the second ball be thrown so that the two balls will splash into the river at the same time? (use $g=-32$ *ft*/sec^2)

A) -40 B) -44 C) -58 D) -80 E) NOTA

16. The amount of a chemical increases at a rate equal to the product of elapsed time (in minutes) and the amount of the chemical. If the initial amount of chemical is 10 units, then what is the number of units at 4 minutes?

A) $10e^4$ B) $10e^8$ C) $10e^{16}$ D) $10e^{32}$ E) NOTA

17. Find the total area of the regions bounded by the graphs of $y=\cos x$ and $y=\cos 2x$ from $x=0$ to $x=\pi$.

A) $1-\frac{\sqrt{3}}{2}$ B) $\sqrt{3}$ C) $\sqrt{3}+\frac{1}{2}$ D) $\frac{3\sqrt{3}}{2}$ E) NOTA

18. Evaluate $\int \cot\left(\frac{x}{5}-1\right)dx$.

A) $5\ln\left|\tan\left(\frac{x}{5}-1\right)\right|+C$ B) $\frac{1}{5}\ln\left|\sin\left(\frac{x}{5}-1\right)\right|+C$ C) $5\ln\left|\cos\left(\frac{x}{5}-1\right)\right|+C$

D) $-5\ln\left|\csc\left(\frac{x}{5}-1\right)\right|+C$ E) NOTA

19. Find $a+b+5c$ if $c^2+a=18$ and $\lim_{x\to\infty}\left(\sqrt{ax^2+bx}-cx\right)=-2$.

A) 18 B) 12 C) 9 D) 5 E) NOTA

20. Use the trapezoidal rule with $n=4$ to approximate the area below the curve $y=2x^2-3x+6$, above the x-axis and between $x=0$ and $x=4$. Find the positive difference between this approximation and the exact area of the region.

A) $\frac{4}{3}$ B) $\frac{5}{3}$ C) $\frac{5}{6}$ D) $\frac{7}{6}$ E) NOTA

21. Given that $f(x)=x^3+x$ and that $g(x)=f^{-1}(x)$, the inverse of $f(x)$, find $f(2)-f'(2)+3f(g(2))-2g(f(2))+5g(2)-4g'(2)$.

A) 2 B) 3 C) 4 D) 5 E) NOTA

22. The sum of two positive numbers is 20. The product of the square of one and the cube of the other is to be made as great as possible. Find the positive difference between the two numbers.

A) $\frac{11}{2}$ B) 5 C) $\frac{9}{2}$ D) 4 E) NOTA

23. If $f(x)=\frac{d}{dx}g(x)$ and both $f(x)$ and $g(x)$ are integrable on $[3,4]$ then evaluate $\int_3^4 f(x)g(x)dx$ given that $f(3)=-6$, $g(3)=-7$, $f(4)=-8$ and $g(4)=-9$.

A) 16 B) 14 C) -14 D) -16 E) NOTA

24. Evaluate $\int_0^5 |x^3 - x|\,dx$.

A) $\frac{577}{4}$ B) $\frac{575}{4}$ C) $\frac{573}{4}$ D) $\frac{571}{4}$ E) NOTA

25. Let C be the curve defined by $x = t^2 + t + 1$ and $y = t^3 - t - 1$. Determine the x-intercept of the line tangent to C at $(3,-1)$.

A) $-\frac{11}{2}$ B) $\frac{9}{2}$ C) $-\frac{7}{2}$ D) $\frac{5}{2}$ E) NOTA

26. Find the volume of the solid generated by revolving the region bounded by $y = 2x^2$, $y = 0$ and $x = 5$ about the line $x = 6$.

A) 375π B) 350π C) 325π D) 300π E) NOTA

27. A right triangle is rotated about one of its legs to generate a cone. If the hypotenuse of the triangle has length k centimeters, then find the length of the radius of the base (in centimeters) of the cone of maximum volume that can be generated.

A) $\frac{k\sqrt{3}}{6}$ B) $\frac{k\sqrt{6}}{3}$ C) $\frac{k\sqrt{3}}{4}$ D) $\frac{k\sqrt{6}}{4}$ E) NOTA

28. Let $f(x) = \frac{1}{3}x^3 - x + 3$ defined on the interval $-2 \le x \le 3$. Which of the following is/are true?

I. An absolute maximum occurs at $x = -1$.
II. A relative minimum occurs at $x = 1$.
III. An inflection point occurs at $x = \frac{1}{2}$.

A) I only B) II only C) I and II only D) II and III only E) NOTA

29. Find $F'(x)$ if $F(x) = \int_x^{\sin x} 2t\,dt$.

A) $2\sin x - 2x$ B) $\sin^2 x - x^2$ C) $2\sin x\cos x - 2$ D) $\sin 2x - 2x$ E) NOTA

30. Find the slope of the line tangent to the graph of $xy^2 + 5\sin x + y = \frac{x}{y^2} + 1$ at the point (0,1).

A) -2 B) -3 C) -4 D) -5 E) NOTA