MAO 2008 – Sequences and Series Alpha Detailed Answers

1) Answer C: To find $a_{10}$, we must plug in the value of 9 for n. Substituting in we get the value of 65

2) Answer B: Using the formula $t_n = t_1 + d(n-1)$, where $t_n$ is the "nth" term, d is the common difference. Plugging in 5 for $t_1$, 90 for $t_n$, and 38 for n results in a common difference of 2.29 which is approximately 2.3

3) Answer C: Plugging in the values of 1-4, results in a sum of .7, which is closest to 2/3

4) Answer A: Using the formula ${}_{n+k-1}C_n = \dfrac{(n+k-1)!}{n!(k-1)!}$ , where n is power raised to, k is the number of distinct terms results in the answer of 36.

5) Answer D: Using the formula $\frac{k}{5^1} + \frac{k}{5^2} + \frac{k}{5^n}$, where $5^n < k$, where k is 1000 in this place. You divide each individual and throw away the remainder. Solving you arrive at an answer of 249

6) Answer B: $pentagonal = \dfrac{n(3n-1)}{2}$; $rec\tan gular = n(n+1)$, $square = n^2$;

$triangular = \dfrac{n(n+1)}{2}$. Subbing in 10 you get a value of 410.

7) Answer A: nth term in kth row of pascal's triangle = ${}_{k-1}C_{n-1}$ = 10626

8) Answer E: Multiply through the sequence and you arrive at $2 + \frac{3}{3} + \frac{4}{9} + \frac{5}{27}$. Subtract this sequence from the first one in order to arrive at a geometric sequence. Solving this equation you arrive at 5/4

9) Answer A: Solving $.12\overline{7} = \dfrac{127-12}{900} = \dfrac{115}{900} = \dfrac{23}{180}$. Add 3 to this and arrive at 563/180

10) Answer A:
$$(1+i)^2 = 2i$$
$$(2i)^{10} = (1+i)^{20} = -1024$$

11) Answer A: If you take 1/7 out, you result in .142857142857. Divide 6 into 103 you have a remainder of 1, which means the $103^{rd}$ digit is 1.

12) Answer A: A harmonic sequence and corresponding arithmetic sequence can not converge, so must always diverge.

13) Answer D: Looking at the last digit, you arrive at a pattern of 3, 9, 7, 1, . . . Dividing 4 into 1333, you arrive at a remainder of 1. So the last digit must be 3.

14) Answer B: When you are looking at convergence, you only care about the highest power of n. A is equivalent to 1/n; B is equivalent to 1/n^1.5, C is equivalent to 1/n^1.5. Looking at these two only B and C converge.

15) Answer A: Plugging in the first few terms of the Lucas Sequence, you are able to derive that
Ln = Fn+1 + Fn-1.

16) Answer C: The second term is derived by subtracting 10, the next by subtracting 8, and so forth. The next two terms in sequence are 22 and 20, which gives a sum of 42.

17) Answer C: Simplifying the infinite sequence you get x = 2+1/x. Solving you get the only usable answer $1+\sqrt{2}$

18) Answer A: Running the program, you get an S value of 2 on the second loop.

19) Answer D: $sum = \frac{n(n+1)(2n+1)}{6} - 2\frac{(n)(n+1)}{2} + 10(n) = 2650$

20) Answer E: The definition of the geometric mean is the square root of the product of two positive numbers. 12*3 = 36. The square root of 36 is 6.

21) Answer A: Max slices = n(n+1)/2 = 300*301/2 = 45150

22) Answer D: The sum of the interior angles = 540. Name the angles 150, 150-r, 150-2r, 150-3r, and 150-4r. Solving the equation you arrive at r = 21. 150-66 = 84

23) Answer C: Making a chart where n = # of strokes. At n = 1, you have 70% remaining. At n = 2, you have 49% remaining. Continuing on, at 7 strokes you fall below 10% at 8.23.

24) Answer C: Looking at the palindromes of the last millennium you have 1001, 1111, 1221, 1331, 1441, 1551, etc. Adding up the 1s digit you get a value of 10.

25) Answer A: sum = cos^2x = cos^4 x+ . . . Since $\left|\cos^{2k} x\right| < 1, sum = \frac{\cos^2 x}{1-\cos^2 x} = \cot^2 x$

26) Answer D: $448 = \frac{n}{2}(2*76 + (n-1)(-4));\ n = 32\ or\ n = 7$

27) Answer B: Rationalizing the first couple of terms shows a pattern. Simplifying you get the answer of $\sqrt{10} - 1$

28) Answer C: Breaking up the fraction, you arrive at $\frac{1}{2n-1}-\frac{1}{2n+1}$. Plugging in the first few terms a pattern emerges. The final result is 1-1/101. This is closest to 0.99.

29) Answer C: Graph the position of each swimmer with respect to time. (use in terms of seconds). At the end of 3 minutes they are back to their original positions, so that in 12 minutes this cycle is repeated 4 times. Since they have 5 meetings in the cycle, the total number of meetings is 20.

30) Answer B: Solving the system:

$$\frac{1}{2}(a+b+c)+d=29;\ \frac{1}{2}(b+c+d)+a=23;$$
$$\frac{1}{2}(c+d+a)+b=21;\ \frac{1}{2}(d+a+b)+c=17$$

A = 12, b = 9, c = 3, d = 21. So the answer is B.