1. B:
Let length $AE = x + 3 \rightarrow 4(4+8) = 3(3+x) \rightarrow x = 13 \rightarrow AE = 16$
Measure of $\angle A = \frac{(CE - BE)}{2} = \frac{(80-20)}{2} = 30°$
Area of $ABE = \frac{1}{2} \cdot 4 \cdot 16 \cdot \sin(30°) = 16$

2. A:

$-16x^2 + 9y^2 + 96x - 54y - 207 = 0 \rightarrow -16(x^2 - 6x + \cdots) + 9(y^2 - 6y + \cdots) = 207$
$-16(x-3)^2 + 9(y-3)^2 = 207 - 16*9 + 9*9 \rightarrow \frac{-(x-3)^2}{9} + \frac{(y-3)^2}{16} = 1$
$c^2 = a^2 + b^2 \rightarrow c = \pm\sqrt{16+9}$
Foci at: $(3, 3 \pm 5)$

3. B:
Foci of the hyperbola at: $(3, 3 \pm \sqrt{(9+16)}$)
Foci of the ellipse at: $\left(3 \pm \sqrt{25-16}, 3\right)$
Distance between foci of hyperbola: $2 * 5$
Distance between foci of ellipse: 2*3
Area = $10 * 6 * \frac{1}{2} = 30$

4. D:
After rotating by 90°, we get: $\frac{(x+4)^2}{4} + \frac{y^2}{9} = 1$
Revolving about *y = 0*, yields an ellipsoid with length axis radius 3, width axis radius 2, and height axis radius 3.
Volume = $3 \times 3 \times 2 \times \frac{4}{3}\pi = 24\pi$

5. A:
Multiply both sides by *r:*
$r^2(9 - 5\sin^2(\theta)) = 36r\cos(\theta) \rightarrow 9(x^2 + y^2) - 5y^2 = 36x \rightarrow 9x^2 + 4y^2 - 36x = 0$
$9 + 4 - 36 = -23$

6. E:

$cis(6\pi + \theta) = \cos(6\pi + \theta) + i\sin(6\pi + \theta) = \cos\theta + i\sin\theta$
$\frac{cis(4\theta)}{cis(3\theta)cis(\theta)} = cis(4\theta - 4\theta) = cis(0) = 1$

7. C:

$\tan x = \frac{1}{a}, \tan y = \frac{1}{b}, \sin x = \frac{1}{c}$

$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1+\cos x} = \frac{1}{\sin x} - \cot x = c - a$

$\cot\left(\frac{x}{2} - y\right) = \frac{1}{\tan\left(\frac{x}{2}-y\right)} = \frac{1+\tan\left(\frac{x}{2}\right)\tan y}{\tan\left(\frac{x}{2}\right) - \tan y} = \frac{1+(c-a)\left(\frac{1}{b}\right)}{c-a-\frac{1}{b}} = \frac{b+c-a}{bc-ab-1}$

8. A:

$\cot 2\theta = \frac{7-13}{A} = \frac{\sqrt{3}}{3}$

$A = -6\sqrt{3}$

9. C:

$p = \frac{a \cdot b}{|a|^2} \cdot a = \frac{4*2+4*5}{4^2+4^2} \cdot (4i+4j) = \frac{7}{2}i + \frac{7}{2}j$

10.B:

$r = 8cos^4\theta - 8cos^2\theta + 1 = 2(4cos^4\theta - 4cos^2\theta + 1) - 1 = 2(2cos^2\theta - 1)^2 - 1$

$r = 2cos^2(2\theta) - 1 = \cos 2(2\theta) = \cos(4\theta) \rightarrow number\ of\ petals = 2*4 = 8$

11. D:

$\sec 2\theta = \frac{x^2+1}{x^2-x} = \frac{1}{1-2sin^2\theta}$

$x^2 - x = x^2 + 1 - 2(x^2+1)sin^2\theta \rightarrow x + 1 = 2(x^2+1)sin^2\theta \rightarrow \sin\theta = \sqrt{\frac{x+1}{2(x^2+1)}}$

12. C:

$r = \pm 5\sqrt{\sec 2\theta} \rightarrow r^2 = \frac{25}{\cos 2\theta} \rightarrow x^2 - y^2 = 25$

Eccentricity = $\frac{\sqrt{25+25}}{\sqrt{25}} = \sqrt{2}$

13. C:

Ellipse has equation: $\frac{x^2}{4} + \frac{y^2}{16} = 1$

c = focal distance = $\sqrt{16-4} = 2\sqrt{3}$

14. D:

Vector $\frac{\sqrt{2}}{2}i+\frac{\sqrt{2}}{2}j$ makes a 45° with the x-axis since $\frac{\cos\pi}{4}=\frac{\sin\pi}{4}=\frac{\sqrt{2}}{2}$. Since the angle between the two vectors is 75, the other vector must make an angle of -30° with the x-axis $\rightarrow \frac{\sqrt{3}}{2}i-\frac{1}{2}j$

15. C:

$$a+bi=\frac{(5+5i)(2-i)}{(2+i)(3+i)}=\frac{15+5i}{5+5i}=2-i$$

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16. C:

Vertex at: $y=-\frac{b}{2a}=-\frac{4}{2}=-2, \qquad x=(-2)^2+4(-2)+5=1$

Parabola opens sideways→ axis of symmetry at *y* = *-2*.

17. B:

Dimpled limaçons have equations of the form $r=a\pm b\sin\theta$ or $r=a\pm b\cos\theta$ for which $a>0, b>0$ and $1<\frac{a}{b}<2$. Therefore B is a dimpled limaçon.

18. A :

$$\coth x=\frac{1}{\tanh x}=\frac{e^x+e^{-x}}{e^x-e^{-x}}=4\rightarrow 3e^x=5e^{-x}\rightarrow x=\frac{1}{2}\ln\frac{5}{3}+k\pi i\rightarrow\frac{1}{2}+\frac{5}{3}=\frac{13}{6}$$

19. E :

$$\arg\left[\sqrt{3}-i\right]=\tan^{-1}\frac{-1}{\sqrt{3}}+2k\pi=-\frac{\pi}{6}+2k\pi, k\in\mathbf{R}$$

20. D :

The slopes of the asymptotes are $\pm\frac{b}{a}$, so the sum will be 0.

21. D:

$$f(x)=\frac{\sqrt{2}}{2}\pi(\cos x-\sin x)-e=\pi\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)-e=\pi\left(\cos x\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\sin x\right)-e$$

$$f(x)=\pi\left(\cos\left(x+\frac{\pi}{4}\right)\right)-e$$

Since cosine can have minimum and maximum values of -1 and 1, respectively, $f(x)$ can attain values of $-\pi-e$ and $\pi-e$. Therefore $a+b=-2e$.

22. A :

$$\cos(4\theta)=(\cos(2\theta)+i\sin(2\theta))^2-i\sin(4\theta)=\cos^2(2\theta)-\sin^2(2\theta)=8\cos^4\theta-8\cos^2\theta+1$$

$$\text{Arcsin}\,(a+b+c)=\text{Arcsin}(1)=\frac{\pi}{2}$$

23. B:

By Green's theorem,

$$Area = \frac{(4*2+2*4+3*5+4y+x-2*1-3*2-4*4-5x-4y)}{2} = \frac{|13|}{2} \rightarrow x = 5, or\ x = -\frac{3}{2}$$

But if $x = -\frac{3}{2}$, the polygon is not convex, so $x = 5$.

To find *y*, plug into equation: $y = 2*5-7 = 3$

$x + y = 8$

24. A:

$f(x) = \frac{1}{\tan^2 x + \sec^2 x + 2\tan x \sec x}$, where tan x and sec x are defined. So there are asymptotes whenever $\tan x + \sec x = 0 \rightarrow x = \frac{3\pi}{2} + 2k\pi, k \in \mathbf{R}$

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25. D :

$$\tan\left(Arctan\left(\frac{1}{2}\right) + Arctan\left(\frac{1}{3}\right)\right) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = 1$$

$Arctan(1) = \frac{\pi}{4}$

26. E :

Period = $LCM\left(\frac{2\pi}{6}, \frac{2\pi}{3}\right) = \frac{2\pi}{3}$

27. B :

$$\frac{512}{2008}\pi = \frac{64}{251}\pi \rightarrow \frac{64}{251}\pi - 2\pi = -\frac{438\pi}{251}$$

28. C

29. B

Although there is a negative sign in front of the *y* term, it is the equation of a hyperbolic paraboloid.

30. A:

$3 + 8 + 2z = 0 \rightarrow z = -\frac{11}{2}$