1. $\boxed{C:64}$ $\frac{dW}{dt}=kW \to \int \frac{dW}{W}=\int kdt \to \ln W = kt + C \to W = Ce^{kt}$ at t = 0, $2 = Ce^0 \therefore C = 2$, so $W = 2e^{kt}$. At t = 3 he has eaten 16 wings, $16 = 2e^{3k} \to e^{3k} = 8 \to k = \frac{1}{3}\ln 8 = \ln 2$. The wing eating function is $W = 2e^{t\ln 2}$, so at t = 5, $W = 2e^{5\ln 2} = 2e^{\ln 32} = 2 \cdot 32 = 64$

2. $\boxed{D:\frac{9\sqrt{3}}{2}}$ If $y^2 = x-3$, then solving for y will give us an upper-bound of $y=\sqrt{x-3}$ and a lower bound of $y=-\sqrt{x-3}$. The side of one of the cross-sections will be $S=\sqrt{x-3}-\left(-\sqrt{x-3}\right) = 2\sqrt{x-3}$ and area of an equilateral triangle is $\frac{s^2\sqrt{3}}{4}$. The volume can be found by $V=\frac{\sqrt{3}}{4}\int_3^6\left(2\sqrt{x-3}\right)^2 dx = \frac{9\sqrt{3}}{2}$.

3. $\boxed{C:5}$ If $g'(c)=\frac{1}{5}$, then $f'(x)=5$ $c=5 \to f'(x)=3x^2+2=5$ solving for x gives us $x=\pm 1$. By plugging both 1 and -1 into f(x) we find that only 1 yields a positive value: 5. If $f(1)=5$, then $D:5$.

4. $\boxed{B:300}$ If $a(t)=-32 \to v(t)=-32t+v_0=-32t-20 \to s(t)=-16t^2-20t+s_0$, then $0=-16t^2-20t+1400 \to 16t^2+20t-1400=4\left(4t^2+5t-350\right)=4\left(4t-35\right)\left(t+10\right)=0$ $t=\frac{35}{4}$. Plugging into the velocity equation gives us $v\left(35/4\right)=-32\left(35/4\right)-20=-280-20=-300$

5. $\boxed{A:\pi}$ Use washer: $V=\pi\int_1^{\infty}\left(\frac{1}{x}\right)^2 dx \to \pi\lim_{b\to\infty}\left[\int_1^b \frac{1}{x^2}dx\right]=\pi\lim_{b\to\infty}\left[-\frac{1}{x}\right]_1^b=\pi\cdot\lim_{b\to\infty}\left(-\frac{1}{b}-\left(-1\right)\right)=\pi$

6. $\boxed{C:\frac{2\pi-3\sqrt{3}}{2}}$ The inner loop of the graph will occur in between the angles when $1-2\sin\theta=0$. $2\sin\theta=1\to$ $\sin\theta=\frac{1}{2}\therefore\theta=\frac{\pi}{6},\frac{5\pi}{6}$ $A=\frac{1}{2}\int_{\pi/6}^{5\pi/6}\left(1-2\sin\theta\right)^2 d\theta = \frac{1}{2}\int_{\pi/6}^{5\pi/6}\left(1-4\sin\theta+4\sin^2\theta\right)d\theta =$

$$\frac{1}{2}\int_{\pi/6}^{5\pi/6}\left(1-4\sin\theta+4\cdot\frac{1-\cos 2\theta}{2}\right)d\theta=\frac{1}{2}\left[3\theta+4\cos\theta-\sin 2\theta\right]_{\pi/6}^{5\pi/6}=\frac{2\pi-3\sqrt{3}}{2}$$

7. $\boxed{B:29}$ $\frac{\int_0^6\left(t^2+7t-4\right)dt}{6-0}=\frac{\left[\frac{t^3}{3}+\frac{7t^2}{2}-4t\right]_0^6}{6}=\frac{72+126-24}{6}=29$

8. $\boxed{C:\dfrac{14\pi e^{3/2}}{3}}$ Use shells- $2\pi\int_0^{\sqrt{3e}} x\sqrt{x^2+e}\,dx \quad u=x^2+e, du=2xdx \to xdx=\dfrac{du}{2}$

$\dfrac{2\pi}{2}\int_e^{4e}\sqrt{u}\,du=\pi\left[\dfrac{2}{3}u^{3/2}\right]_e^{4e}=\dfrac{2\pi}{3}\left[8e^{3/2}-e^{3/2}\right]=\dfrac{14\pi e^{3/2}}{3}$

9. $\boxed{A:(0,1)\cup(2,\infty)}$ If $A:(0,1)\cup(2,\infty)$, then $v(t)=3t^2-6t$. Velocity will be negative on the interval $(0,2)$ and positive $(2,\infty)$. $a(t)=6x-6$ and changes from negative to positive at t = 1. Speed is increasing when velocity and acceleration have the same sign. Velocity and acceleration are both negative from $(0,1)$ and both positive from $(2,\infty)$.

10. $\boxed{D:\pm\dfrac{4}{15}}$ The maximum possible error diameter within the acceptable parameters will occur when the maximum error occurs in the opposite direction for the height. If the height has a negative error, the diameter may have a larger error and still meet the volume requirements. $V_{cone}=\dfrac{\pi r^2 h}{3}\to dV=\dfrac{2}{3}\pi rhdr+\dfrac{1}{3}\pi r^2 dh$. The maximum positive error in volume, $\dfrac{\pi}{6}$, and maximum negative error in height, $-\dfrac{2}{3}$, will give us the maximum possible error in diameter. $dV=\dfrac{2}{3}\pi rhdr+\dfrac{1}{3}\pi r^2dh\to\dfrac{\pi}{6}=\dfrac{2}{3}\pi\left(\dfrac{3}{2}\right)(5)\,dr+\dfrac{1}{3}\pi\left(\dfrac{3}{2}\right)^2\left(-\dfrac{2}{3}\right)$. Solving for dr, we find $dr=\dfrac{2}{15}$ therefore the error in diameter will be $\pm\dfrac{4}{15}$ depending on which values are used for dv and dh.

11. $\boxed{B:\left(1,-\dfrac{3}{5}\right)}$ Area of the region is $\int_0^2\left(2x-x^2\right)dx=\dfrac{4}{3}$ $\quad \bar{x}=\dfrac{1}{A}\int_a^b x\left[f(x)-g(x)\right]dx=\dfrac{3}{4}\int_0^2 x\left[2x-x^2\right]dx=$

$\dfrac{3}{4}\left[\dfrac{2x^3}{3}-\dfrac{x^4}{4}\right]_0^2=1 \quad \bar{y}=\dfrac{1}{2A}\int_a^b\left[\left(f(x)\right)^2-\left(g(x)\right)^2\right]dx=\dfrac{3}{8}\int_0^2\left[\left(x-x^2\right)^2-(-x)^2\right]dx=\dfrac{3}{8}\left[-\dfrac{x^4}{2}+\dfrac{x^5}{5}\right]_0^2=-\dfrac{3}{5}$.

12. $\boxed{E:2\sqrt{10}}$ Total acceleration will be $\sqrt{\dfrac{d^2x}{dt^2}+\dfrac{d^2y}{dt^2}}=\sqrt{(3t)^2+\left(-\dfrac{8}{t^2}\right)}$. At t = 2 $\sqrt{(6)^2+(-2)^2}=2\sqrt{10}$

13. $\boxed{C:\dfrac{7}{4}}$ The distance between the foci is 2c from the equation $a^2-b^2=c^2\to 2a\dfrac{da}{dt}-2b\dfrac{db}{dt}=2c\dfrac{dc}{dt}$ and the rate of change of the distance will be $2\dfrac{dc}{dt}$. If $\dfrac{x^2}{25}+\dfrac{y^2}{9}=1$, then a = 5, b= 3 and c will equal 4. a and b will decrease at half the of rate of their respective axes. $2a\dfrac{da}{dt}-2b\dfrac{db}{dt}=2c\dfrac{dc}{dt}\to 5\left(\dfrac{1}{2}\right)-3\left(-\dfrac{1}{3}\right)=4\dfrac{dc}{dt}$. Solving for $\dfrac{dc}{dt}=\dfrac{7}{8}$, so the rate of change is $\dfrac{7}{4}$.

14. $\boxed{D:\frac{128\pi}{5}}$ Use shells - $V=2\pi\int_0^4(6-x)\left(2-\sqrt{x}\right)dx=2\pi\left[12x-x^2-2x^{3/2}+2/5\,x^{5/2}\right]_0^4=\frac{128\pi}{5}$

15. $\boxed{B:\text{MVT for D}}$ The MVT for Derivatives states that for a continuous, differentiable function over a given interval there must be at least one point in between the endpoints that the rate of change at that point must be equal to average rate between the endpoints. In this case the care goes 30 miles in .5 hours giving an average rate of change of 60 mph.

16. $\boxed{A:e^{\frac{(\ln x)^2+1}{2}}}$ $\ln x\frac{dx}{dy}=\frac{x}{y}$ $\frac{\ln x}{x}dx=\frac{dy}{y}\to\int\frac{\ln x}{x}dx=\int\frac{dy}{y}\to\frac{(\ln x)^2}{2}+C=\ln y\to y=Ce^{\frac{(\ln x)^2}{2}}$ If $y\left(e^3\right)=e^5$, then $e^5=Ce^{\frac{(\ln e^3)^2}{2}}=Ce^{\frac{9}{2}}\ \therefore C=e^{\frac{1}{2}}$ $y=e^{\frac{1}{2}}\cdot e^{\frac{(\ln x)^2}{2}}=e^{\frac{(\ln x)^2+1}{2}}$

17. $\boxed{C:2\pi\int_0^1 x\cdot\sqrt{1+x^2}\,dx}$ $SA=2\pi\int_a^b r(x)\cdot\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ As we are revolving about the y-axis, the radius will be x and $\frac{d}{dx}\left[\frac{x^2+1}{2}\right]=x$ $2\pi\int_0^1 x\cdot\sqrt{1+x^2}\,dx$

18. $\boxed{C:3}$ $\frac{\int_0^b\left(1+6x-3x^2\right)dx}{b}=3\to\frac{\left[x+3x^2-x^3\right]_0^b}{b}=3\to b+3b^2-b^3=3b\quad\to b^3-3b^2+2b=0$

$\to b(b-2)(b-1)=0\to b=0,1,2$ b cannot be zero sum of 1 and 2 is 3

19. $\boxed{D:\frac{71}{6}}$ The curves intersect at x = -3,0,1 $A=\int_{-3}^0\left(x^3+2x^2-3x\right)dx+\int_0^1\left(3x-\left(x^3+2x^2\right)\right)dx=\frac{71}{6}$

20. $\boxed{B:6\sqrt{3}}$ The area function for the triangle will be $A(x)=\frac{1}{2}(2x)\left(9-x^2\right)=9x-x^3$ $A'(x)=9-3x^2=0$ $\to x=\pm\sqrt{3}$ Plugging into the area function we will get $9\sqrt{3}-3\sqrt{3}=6\sqrt{3}$.

21. $\boxed{A:\frac{\pi}{6}}$ If $T=2\pi\sqrt{\frac{L}{g}}$, then $\frac{dT}{dt}=\frac{2\pi}{\sqrt{g}}\cdot\frac{1}{2\sqrt{L}}\cdot\frac{dL}{dt}$. L = 9, g = 16, $\frac{dL}{dt}=2$. We get $\frac{\pi}{6}$.

22. $\boxed{E:y-\frac{\pi}{3}=-4\left(x-\sqrt{3}\right)}$ $y=\arctan x$ $y'(x)=\frac{1}{1+x^2}$ $m=\frac{1}{4}\to m_\perp=-4$. For $x=\sqrt{3},y=\frac{\pi}{3}$. The normal line is $y-\frac{\pi}{3}=-4\left(x-\sqrt{3}\right)$

23. $\boxed{B:19600}$ $F=mg=10kg\cdot 9.8\,m/s^2=98\,Newtons$ $W=\int_0^{20}F(x)dx=\int_0^{20}98x\,dx=\left[49x^2\right]_0^{20}=19600\,joules$

24. $\boxed{D: \frac{7}{3}}$ Max slope will be 2nd der. set equal to 0. $f'(x) = -3x^2 + 4x + 1 \rightarrow f''(x) = -6x + 4 = 0$

$x = \frac{2}{3}$. $f'''(x)$ is negative for all x so $x = \frac{2}{3}$ will be a maximum for slope. $f'\left(\frac{2}{3}\right) = -3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) + 1 = \frac{7}{3}$

25. $\boxed{D: 180\pi^2}$ Pappus $\rightarrow V = 2\pi R\left(Area\right)$ The equation is a circle with radius of 3 and center (4,-5) so the area of the region bounded is 9π. The distance between can be found by find the distance from the center of the circle and the point of intersection of the line and it's normal that includes the center. The line is $4x - 3y = 19$ and the normal is $3x + 4y = -8$. Solving the system gives us the point (-4,1). The distance between (-4,1) and (4,-5) is 10. The volume of the solid will be $180\pi^2$.

26. $\boxed{C: \left\langle 8\sqrt{3}, -\frac{18}{\pi^2} \right\rangle}$ $i''(t) = 2\sec^2 t \tan t$, $j''(t) = -\frac{2}{t^2}$, $i''\left(\pi/3\right) = 2\sec^2\left(\pi/3\right)\tan\left(\pi/3\right) = 8\sqrt{3}$

$j''\left(\pi/3\right) = -\frac{2}{\left(\pi/3\right)^2} = -\frac{18}{\pi^2}$ So the vector will be $\left\langle 8\sqrt{3}, -\frac{18}{\pi^2} \right\rangle$

27. $\boxed{A: 45^\circ}$ Setting f(x) and g(x) equal we find that they intersect at x = 3, 4/3. We want the greatest value of x so we use x = 3. The slope of f(x) at 3 is $f'(3) = 4(3) - 10 = 2$ and slope of g(x) is $g'(3) = -2(3) + 3 = -3$. The tangent of the angle between two slopes can be found with $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$. $\tan\theta = \frac{2 - (-3)}{1 + (2)(-3)} = -\frac{5}{5} = -1$.

$\arctan(-1) = 135^o$ so the smaller angle would be 45°

28. $\boxed{C: 216}$ $V = \pi r^2 h \rightarrow 36\cancel{\pi} = \cancel{\pi} r^2 h \rightarrow h = \frac{36}{r^2}$ $SA = 2\pi r^2 + 2\pi rh \rightarrow$

$\rightarrow Weight = 2\cancel{\pi} r^2\left(\frac{9}{\cancel{\pi}}\right) + 2\cancel{\pi} rh\left(\frac{4}{\cancel{\pi}}\right) \rightarrow 2r^2 \cdot 9 + 2r\left(\frac{36}{r^2}\right) \cdot 4 = 18r^2 + \frac{288}{r}$. Min. weight $W' = 36r - \frac{288}{r^2} = 0$

Solving for r we get r = 2. Plugging into the weight function we get $W(2) = 18(2)^2 + \frac{288}{2} = 216$

29. $\boxed{B: \frac{1}{2}\ln\left(2+\sqrt{3}\right)}$ Arclength = $\int_\alpha^\beta \sqrt{\left(dx/dt\right)^2 + \left(dy/dt\right)^2}\,dt = \frac{1}{2}\int_0^{\pi/3} \sec t\,dt = \frac{1}{2}\left[\ln\left(\sec t + \tan t\right)\right]_0^{\pi/3}$

$\rightarrow \int_0^{\pi/3} \sqrt{\left(-\tan t/2\right)^2 + \left(1/2\right)^2}\,dt = \frac{1}{2}\int_0^{\pi/3} \sqrt{\tan^2 t + 1}\,dt = \frac{1}{2}\int_0^{\pi/3} \sec t\,dt = \frac{1}{2}\left[\ln\left(\sec t + \tan t\right)\right]_0^{\pi/3} = \frac{1}{2}\ln\left(2+\sqrt{3}\right)$

30. $\boxed{C: 34}$ Marginal cost will be the derivative of the cost function $C(x) = x \cdot p(x) - r(x)$ where $C(x)$ is the cost function, $r(x)$ is the profit function and $p(x)$ is the price per Snitch. $C(x) = x \cdot \frac{x^2 - 4x + 9}{2} - \left(1/2\, x + 4\right)$

$= \frac{1}{2}\left(x^3 - 4x^2 + 9x\right) - \left(\frac{1}{2}x + 4\right) \quad C'(x) = \frac{1}{2}\left(3x^2 - 8x + 9\right) - \left(\frac{1}{2}\right) = \frac{3}{2}x^2 - 4x + 4$

$C'(6) = \frac{3}{2} \cdot 6^2 - 4 \cdot 6 + 4 = 34$

Answers:

1. C
2. D
3. C
4. B
5. A
6. C
7. B
8. C
9. A
10. D
11. B
12. E
13. C
14. D
15. B
16. A
17. C
18. C
19. D
20. B
21. A
22. E
23. B
24. D
25. D
26. C
27. A
28. C
29. B
30. C