

Mu Alpha Theta 2008 National Convention
Mu Applications

1. Joe is in a chicken wing eating contest and the rate at which he eats chicken wings is directly proportional to the amount of chicken wings he has eaten. Before the contest starts ($t = 0$) he has eaten 2 chicken wings to get his “stomach started”. If Joe has eaten 16 wings after 3 minutes (not including the 2 he ate before the contest started), how many wings will he have eaten after 5 minutes? (Do not include the wings eaten before the contest.)
- A. 24 B. 32 C. 64 D. 96 E. NOTA
2. Find the volume of a solid that has known cross-sections of equilateral triangles with a bases lying perpendicular to the x-axis in region bounded by $y^2 = x - 3$ and $x = 6$.
- A. $\frac{9\sqrt{3}}{8}$ B. 3 C. $2\sqrt{3}$ D. $\frac{9\sqrt{3}}{2}$ E. NOTA
3. If $g(x) = f^{-1}(x)$ and $f(x) = x^3 + 2x + 2$ find the positive x-coordinate, c , of $g(x)$ for which $g'(c) = \frac{1}{5}$.
- A. -1 B. 1 C. 5 D. 11 E. NOTA
4. Keith has suffered an incredible loss with the recent passing of his pet rock “Jordan”. Jordan’s final wish was to be dropped from the edge of the Grand Canyon. If Keith throws the rock downward in despair at a speed of $20 \frac{ft}{sec}$, what will the velocity of Jordan the Rock be, in feet per second, the instant before it hits the ground 1400 ft below? (Assume $a = -32 \frac{ft}{sec^2}$)
- A. -280 B. -300 C. -340 D. -400 E. NOTA
5. The solid found by revolving the unbounded region lying between the graph of $f(x) = \frac{1}{x}$ and the x-axis ($x \geq 1$) about the x-axis is known as Gabriel’s Horn. Find the volume of Gabriel’s Horn.
- A. π B. $\frac{3\pi}{2}$ C. 2π D. Divergent E. NOTA
6. Find the area of the inner loop of the graph of $r = 1 - 2 \sin \theta$.
- A. $\pi - 2\sqrt{3}$ B. $\frac{\pi - 4\sqrt{3}}{3}$ C. $\frac{2\pi - 3\sqrt{3}}{2}$ D. $\frac{5\pi - 3\sqrt{3}}{2}$ E. NOTA

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7. James has become widely regarded for his drawings of hippopotarats (A cross-breed of a hippopotamus and rat). If the number of drawings per day that James draws can be described by the function $d(t) = t^2 + 7t - 4$ where t is the time in days, find the average number of drawings James makes per day from 0 to 6 days.
- A. 23 B. 29 C. 145 D. 174 E. NOTA
8. Find the volume of the solid formed when the region bounded by the curves $y = \sqrt{x^2 + e}$, $x = 0$, $x = \sqrt{3e}$ and $y = 0$ is revolved about the y -axis.
- A. $\frac{7\pi e^{3/2}}{6}$ B. $\frac{7\pi e^{3/2}}{3}$ C. $\frac{14\pi e^{3/2}}{3}$ D. $7\pi e^{3/2}$ E. NOTA
9. The position of a particle traveling with respect to time along a horizontal line can be described by the function $s(t) = t^3 - 3t^2 + 2$. On what intervals will the speed of the particle be increasing?
- A. $(0,1) \cup (2,\infty)$ B. $(0,1)$ C. $(1,\infty)$ D. $(2,\infty)$ E. NOTA
10. In the wonderful world of waffle-cones, the Acme waffle-cone machine is attempting to make cones that have a diameter of 3 inches and a height of 5 inches. If the maximum allowable error in volume of the cone is $\pm \frac{\pi}{6} \text{ in}^3$ and the maximum error in the height of the cone is $\pm \frac{2}{3} \text{ in}$, what is the maximum error in the diameter of the cone?
- A. $\pm \frac{13}{180}$ B. $\pm \frac{2}{15}$ C. $\pm \frac{13}{90}$ D. $\pm \frac{4}{15}$ E. NOTA
11. Find the centroid of the region bounded by $y = x - x^2$ and the line $x + y = 0$.
- A. $\left(1, -\frac{6}{5}\right)$ B. $\left(1, -\frac{3}{5}\right)$ C. $\left(\frac{1}{2}, -\frac{6}{5}\right)$ D. $\left(\frac{1}{2}, -\frac{2}{5}\right)$ E. NOTA
12. Ricky is taking a paddleboat ride into a lake for a romantic evening with a special someone. His position in the lake can be described by the parametric equations $x = \frac{1}{2}t^3$, $y = 8\ln t$. Find the magnitude of Ricky's total acceleration at $t = 2$.
- A. 1 B. $2\sqrt{5}$ C. $2\sqrt{13}$ D. $4\sqrt{10}$ E. NOTA

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13. At the instant the length of the major axis of an ellipse is increasing at a rate of $1 \frac{\text{unit}}{\text{sec}}$ and the minor axis is decreasing at a rate of $\frac{2}{3} \frac{\text{unit}}{\text{sec}}$, the ellipse has equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$. At what rate is the distance between the foci changing at this time?
- A. $\frac{3}{8}$ B. $\frac{5}{8}$ C. $\frac{7}{4}$ D. $\frac{3}{2}$ E. NOTA
14. Find the volume of the solid formed by revolving the region bounded by $y = 2$, $y = \sqrt{x}$ and the y -axis about the line $x = 6$.
- A. $\frac{64\pi}{15}$ B. $\frac{48\pi}{5}$ C. $\frac{64\pi}{5}$ D. $\frac{128\pi}{5}$ E. NOTA
15. A car's position can be described by a continuous differentiable function. The path the car takes has a speed limit of 50 mph. If the car passes a police car going 45 mph and 30 minutes later passes another police car 30 miles away going 40 mph, what mathematical reasoning could the police offer use to prove that the car exceeded the speed limit at least once over the interval?
- A. L'Hopital's Rule B. Mean Value Theorem for Derivatives
C. Intermediate Value Theorem D. The Squeeze Theorem E. NOTA
16. Find the particular solution to the differential equation $\ln x \frac{dx}{dy} = \frac{x}{y}$, given $y(e^3) = e^5$.
- A. $y = e^{\frac{(\ln x)^2 + 1}{2}}$ B. $y = e^{\frac{(\ln x)^2 + 2}{2}}$ C. $y = e^{(\ln x)^2}$ D. $y = e^{(\ln x) + 1}$ E. NOTA
17. Which of the following integrals expresses the surface area of solid formed when the curve $y = \frac{x^2 + 1}{2}$ on the interval $0 \leq x \leq 1$ is revolved about the y -axis?
- A. $\pi \int_0^1 (x^2 + 1)^{3/2} dx$ B. $\frac{\pi}{2} \int_0^1 (x^2 + 1)^{3/2} dx$ C. $2\pi \int_0^1 x \cdot \sqrt{1 + x^2} dx$
D. $\pi \int_0^1 x \sqrt{1 + x^2} dx$ E. NOTA
18. Find the sum of the possible values of b such that the average value of $f(x) = 1 + 6x - 3x^2$ on the interval on the interval $[0, b]$ is equal to 3.
- A. 1 B. 2 C. 3 D. 5 E. NOTA

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19. Find the area of the region bounded by the curves $y = 3x$ and $y = x^3 + 2x^2$.
- A. $\frac{21}{2}$ B. $\frac{32}{3}$ C. $\frac{45}{4}$ D. $\frac{71}{6}$ E. NOTA
20. Find the maximum area of a triangle that with one vertex at the origin and the other two vertices lying on the curve $y = 9 - x^2$ and having equal positive y -values.
- A. $2\sqrt{6}$ B. $6\sqrt{3}$ C. $12\sqrt{3}$ D. $12\sqrt{6}$ E. NOTA
21. The time period of a pendulum, which is how long it takes a pendulum to complete one cycle, can be expressed by the equation $T = 2\pi\sqrt{\frac{L}{g}}$, where T is the time in seconds, L is the length in centimeters and g is the gravitational field strength. On the planet Schrute, $g = 16\frac{m}{s^2}$. Find the rate at which the Period is changing if a pendulum $9m$ long is increasing in length at a rate of $2\frac{m}{sec}$.
- A. $\frac{\pi}{6}$ B. $\frac{2\pi}{9}$ C. $\frac{\pi}{3}$ D. $\frac{4\pi}{9}$ E. NOTA
22. Find the equation of the line normal to the curve $y = \arctan x$ at $x = \sqrt{3}$.
- A. $y - \frac{\pi}{3} = \frac{1}{4}(x - \sqrt{3})$ B. $y - \frac{\pi}{6} = \frac{1}{4}(x - \sqrt{3})$ C. $y - \frac{\pi}{3} = \frac{1}{2}(x - \sqrt{3})$
- D. $y - \frac{\pi}{3} = 4(x - \sqrt{3})$ E. NOTA
23. A wench is winding up a 20 meter chain with a mass of $10\text{ kg}/m$ that is fully extended. Find the work in joules done by the wench to wind up the entire chain. (Assume acceleration due to gravity is $9.8\text{ m}/s^2$)
- A. 5200 B. 19600 C. 24600 D. 39200 E. NOTA
24. What is the maximum slope of the curve $y = -x^3 + 2x^2 + x - 3$?
- A. $-\frac{67}{27}$ B. $-\frac{47}{27}$ C. -1 D. $\frac{7}{3}$ E. NOTA
25. Find the volume of the solid formed when the region bounded by $x^2 + y^2 - 8x + 10y + 32 = 0$ is revolved about the line $4x - 3y = -19$.
- A. $27\pi^2$ B. 180π C. $90\pi^2$ D. $180\pi^2$ E. NOTA

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26. Find the acceleration vector of a particle traveling with a position vector $\langle \tan t, \ln(t^2) \rangle$ at $t = \frac{\pi}{3}$.
- A. $\langle 2\sqrt{3}, -\frac{18}{\pi^2} \rangle$ B. $\langle 4, \frac{6}{\pi} \rangle$ C. $\langle 8\sqrt{3}, -\frac{18}{\pi^2} \rangle$ D. $\langle 4\sqrt{3}, -\frac{18}{\pi^2} \rangle$ E. NOTA
27. Find the smaller angle in degrees between the tangent lines of $f(x) = 2x^2 - 10x + 7$ and $g(x) = -x^2 + 3x - 5$ at their point of intersection in which x has the greatest value.
- A. 45° B. 60° C. 135° D. 150° E. NOTA
28. In an effort to ship more oil on a freighter and therefore, save money on oil, the Acme Oil Company is trying to make a cylindrical oil tank (with closed ends) that must hold 36π cubic feet of oil and has minimum weight. If the weight of the material for the lateral area of the tank is $\frac{4 \text{ lb}}{\pi \text{ ft}^2}$ and the weight of the material for the bases is $\frac{9 \text{ lb}}{\pi \text{ ft}^2}$. Find the minimum weight of an empty tank in pounds.
- A. 108 B. 144 C. 216 D. 288 E. NOTA
29. Find the length of the parametric curve defined by $x = \frac{\ln(\cos t)}{2}$ and $y = \frac{1}{2}t + 3$ from $0 \leq t \leq \frac{\pi}{3}$.
- A. $\frac{1}{2} \ln\left(\frac{6 + \sqrt{3}}{3}\right)$ B. $\frac{1}{2} \ln(2 + \sqrt{3})$ C. $\ln(2 + \sqrt{3})$ D. $\frac{1}{2} \ln\left(\frac{5\sqrt{3}}{3}\right)$ E. NOTA
30. The price of a Golden Snitch in Galleons can be described by the function $p(x) = \frac{x^2 - 4x + 9}{2}$ where x is the number of Snitches produced. If the total profit in Galleons for making x Golden Snitches can be described by the function $r(x) = \frac{1}{2}x + 4$, what is the marginal cost, in Galleons, for producing 6 Golden Snitches.
- A. 3.5 B. 17 C. 34 D. 76 E. NOTA