1) A: To find $a_{10}$, we must plug in the value of 9 for n. Substituting in we get the value of 742.

2) A: Multiply through the sequence by 3 and you arrive at $2-\frac{3}{3}+\frac{4}{9}-\frac{5}{27}\ldots$. Subtract this sequence from the first one in order to arrive at a geometric sequence: 2n = 2 – 1/3 + 1/9 – 1/27 = . . . Solving this equation you arrive at 7/8.

3) B: multiplying by 100 and subtracting the two equations: 100n = 375.7575… and n = 3.757575. Subtracting gives 99n = 372; n = 372/99; n = 124/33

4) A: $(1+i)^2 = 2i; (2i)^{10} = -1024$

5) D: Looking at the last digit, you arrive at a pattern of 8, 4, 2, 6, . . . Dividing 4 into 2008, you arrive at a remainder of 0. So the last digit must be 6.

6) C: Simplifying the infinite sequence you get x = 2+1/x. Solving you get the only usable answer $1+\sqrt{2}$

7) D: $sum = \frac{n(n+1)(2n+1)}{6} - 2\frac{(n)(n+1)}{2} + 10(n)$ = 2650

8) B: Rationalizing the first couple of terms shows a pattern. Simplifying you get the answer of $\sqrt{10}-1$

9) A: Using partial fractions to break up the fraction, you arrive at $1.5(\frac{1}{2n-1}-\frac{1}{2n+1})$. Plugging in the first few terms a pattern emerges. By plugging in all of the terms we get additive inverses for all but the first 1/5. (1.5)(0.2) = 0.3

10) C: $\sum_{n=0}^{\infty}\tan^n x$ is a geometric series with ratio $\tan x$. It will converge if $|\sin x| < |\cos x|$. This happens when $0 \le x < \pi/4$ or $3\pi/4 < x \le \pi$.

11) D: Call the terms: $8, x, x^2$. S = 8 + x + x²; $S' = 8 + 16x$; $x = -.5$ gives the minimum value of S. S(-0.5) = 6. ,

12) A: Method 1: $f\left(\frac{1}{3}\right) = 1+\frac{2}{3}+\frac{3}{9}+\frac{4}{27}+\cdots$ and $3f\left(\frac{1}{3}\right) = 5+\frac{3}{3}+\frac{4}{9}+\frac{5}{27}+\cdots$

Then $3f\left(\frac{1}{3}\right) - f\left(\frac{1}{3}\right) = 2f\left(\frac{1}{3}\right) = 4+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots = 4+\frac{1/3}{1-1/3} = \frac{9}{2}$. So $f\left(\frac{1}{3}\right) = \frac{9}{4}$.

Method 2: $f(x) = 1+2x+3x^2+\cdots \Rightarrow \int f(x)dx = x+x^2+\cdots = \frac{x}{1-x}$

$$f(x) = \frac{d}{dx}\left(\frac{x}{1-x}\right) = \frac{1}{(1-x)^2} \Rightarrow f\left(\frac{1}{3}\right) = \frac{9}{4}$$

13) C: A. converges by p-series test.

B. converges by limit comparison test.

$\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges, so since $\lim_{n\to\infty}\frac{1}{n^2-1}\cdot\frac{n^2}{1} = 1$, $\sum_{n=1}^{\infty}\frac{1}{n^2-1}$ converges.

C. diverges by comparison test.

$\frac{1}{n^2-1}>\frac{1}{n^2}$, so $\frac{n}{n^2-1}>\frac{n}{n^2}=\frac{1}{n}$. $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, so $\sum_{n=1}^{\infty}\frac{n}{n^2-1}$ diverges.

D. converges by comparison test.

$\frac{1}{n^4+1}<\frac{1}{n^4}$, so $\frac{n}{n^4+1}<\frac{n}{n^4}=\frac{1}{n^3}$. $\sum_{n=1}^{\infty}\frac{1}{n^3}$ converges, so $\sum_{n=1}^{\infty}\frac{n}{n^4+1}$ converges.

14) A: $\sum_{n=2}^{\infty}\frac{1}{x-x^2}=\sum_{n=2}^{\infty}\left(\frac{1}{x}-\frac{1}{x-1}\right)=\left(\frac{1}{2}-1\right)+\left(\frac{1}{3}-\frac{1}{2}\right)+\left(\frac{1}{4}-\frac{1}{3}\right)+\cdots=-1$

15) C:

$$L=\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=\lim_{n\to\infty}\left(\frac{a_{n+2}}{a_{n+1}}\right)=\lim_{n\to\infty}\left(\frac{a_n+2a_{n+1}}{a_{n+1}}\right)=\lim_{n\to\infty}\frac{a_n}{a_{n+1}}+\lim_{n\to\infty}\frac{2a_{n+1}}{a_{n+1}}=\frac{1}{L}+2$$

So $L=\frac{1}{L}+2\Rightarrow L^2-2L-1$. So $L=1-\sqrt{2}$ and $L=1+\sqrt{2}$.

Clearly, L must be positive. So $L=1+\sqrt{2}$.

16)B: In an isosceles right triangle of leg-length 6, the altitude to the hypotenuse has a length of $3\sqrt{2}$, which gives the new, smaller triangle an altitude length to hypotenuse of 3 and so on. This forms an infinite geometric series with first term $3\sqrt{2}$ and ratio of $\frac{1}{\sqrt{2}}$. $S=\frac{3\sqrt{2}}{1-1/\sqrt{2}}=\frac{3\sqrt{2}\cdot\sqrt{2}}{\sqrt{2}-1}=\frac{6}{\sqrt{2}-1}=6\sqrt{2}+6$

17) B: The 4th degree term centered at x = 2 will be $\frac{f^4(x)(x-2)^4}{4!}=\frac{-6/x^4\,(x-2)^4}{4!}$. Plug in 2 for x

$\frac{-6}{2^4\cdot 4!}=\frac{-1}{16\cdot 4}=-\frac{1}{64}$.

18) D: $\lim_{n\to\infty}\frac{1+8+27+...+n^3}{n^4}=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^3}{n^4}=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^3\frac{1}{n}=\int_0^1 x^3dx$

19) C: $\sum_{n=1}^{\infty}\frac{2n}{5^n}=\frac{2}{5}+\frac{4}{25}+\frac{6}{125}+\frac{8}{625}=x \quad 5x=2+\frac{4}{5}+\frac{6}{25}+\frac{8}{125}...$

$$5x-x=\left(2+\frac{4}{5}+\frac{6}{25}+\frac{8}{125}...\right)-\left(\frac{2}{5}+\frac{4}{25}+\frac{6}{125}+\frac{8}{625}...\right)\to 4x=2+\frac{2}{5}+\frac{2}{25}+...\to 4x=\frac{2}{1-1/5}=\frac{5}{2}\quad x=\frac{5}{8}$$

20) A: Use ratio test $\lim_{n\to\infty}\left|\dfrac{\dfrac{(-1)^{n+2}(x-2)^{n+1}}{(n+1)2^{n+1}}}{\dfrac{(-1)^{n+1}(x-2)^{n}}{n\cdot 2^{n}}}\right| = \left|\dfrac{\dfrac{(x-2)}{2(n+1)}}{\dfrac{1}{n}}\right| = \left|\dfrac{(x-2)n}{2(n+1)}\right| = \left|\dfrac{x-2}{2}\right| < 1 \quad |x-2|<2 \to 0<x<4$. Check endpoints $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}(-2)^n}{n\cdot 2^n} = \dfrac{(-1)^{n+1}(-1)^n}{n} = \dfrac{(-1)^{2n+1}}{n}$ diverges. $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}(2)^n}{n\cdot 2^n} = \dfrac{(-1)^{n+1}}{n}$ alternating series converges. $0<x\le 4$

21) C: I) Integral test applies – continuous, positive and decreasing over entire interval

II) Does not apply – $\dfrac{\arctan(0)}{0^2+1}=0 \quad \dfrac{\arctan(1)}{1^2+1}=\dfrac{\pi}{8}$, therefore the series increasing on the interval

III) Applies - $y=e^{2x-x^2}$ achieves a maximum at x=1 and $\dfrac{dy}{dx}$ is negative when $x>1$, therefore decreasing.

IV) Does not apply – when x>1 the numerator will have a negative value, so the terms will be negative.

II,IV do not apply

22) E: $y=\sqrt{x^2+y} \to y^2=x^2+y \to 2y\dfrac{dy}{dx}=2x+\dfrac{dy}{dx} \qquad 2y\dfrac{dy}{dx}-\dfrac{dy}{dx}=2x \to \dfrac{dy}{dx}(2y-1)=2x \to \dfrac{dy}{dx}=\dfrac{2x}{2y-1}$

23) B: $3^{1/2}, 3^{3/4}, 3^{7/8} \ldots 3^{\frac{2^n-1}{2^n}} \quad \lim_{n\to\infty}\dfrac{2^n-1}{2^n}=1 \therefore 3^1=3$

24) A: $\lim_{n\to\infty}\left|\dfrac{\dfrac{(2n+2)!x^{n+1}}{((n+1)!)^2}}{\dfrac{(2n)!x^n}{(n!)^2}}\right| = \left|\dfrac{(2n+2)(2n+1)x}{(n+1)^2}\right| = \left|\dfrac{x(4n^2+6n+2)}{n^2+2n+1}\right| = |4x| \quad |4x|<1 \therefore |x|<\dfrac{1}{4}$

25) C: This series can be broken into three series. I) $1+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}.. = \dfrac{1}{1-1/2}=2$ II) $\dfrac{1}{3}+\dfrac{1}{9}+\dfrac{1}{27}\ldots = \dfrac{1/3}{1-1/3}=\dfrac{1}{2}$

III) $\dfrac{1}{6}+\dfrac{1}{12}+\dfrac{1}{24}\ldots+\dfrac{1}{18}+\dfrac{1}{36}+\dfrac{1}{72}\ldots+\dfrac{1}{54}+\dfrac{1}{108}+\dfrac{1}{216}\ldots =$

$\dfrac{1}{3}\sum_{n=1}^{\infty}\dfrac{1}{2^n}+\dfrac{1}{9}\sum_{n=1}^{\infty}\dfrac{1}{2^n}+\dfrac{1}{27}\sum_{n=1}^{\infty}\dfrac{1}{2^n}\ldots = \sum_{n=1}^{\infty}\dfrac{1}{2^n}\cdot\left[\dfrac{1}{3}+\dfrac{1}{9}+\dfrac{1}{27}\ldots\right] = 1\cdot\dfrac{1}{2}=\dfrac{1}{2}$ I + II + III = $2+\dfrac{1}{2}+\dfrac{1}{2}=3$

26) B: $P(x)=e^0+e^o x+\dfrac{e^0x^2}{2}+\dfrac{e^0x^3}{6}+\dfrac{e^0x^4}{24}=1+x+\dfrac{x^2}{2}+\dfrac{x^3}{6}+\dfrac{x^4}{24} \quad P(1)=1+1+\dfrac{1^2}{2}+\dfrac{1^3}{6}+\dfrac{1^4}{24}=\dfrac{65}{24}$

27) B: $|\sin n|\le 1$ use comparison test to compare to $\dfrac{1}{n^2}$. Since $\left|\dfrac{\sin n}{n^2}\right|\le\dfrac{1}{n^2}$ the series is absolutely convergent.

28) A: Knowing that $\sum_{n=0}^{\infty}\dfrac{x^n}{n!}=e^x$ We have the equation $e^{2x-1}=6 \to \ln 6=2x-1 \to 2x=\ln 6+1 \to x=\ln\sqrt{6e}$.

29) D: $\lim_{x\to\infty}\left|\dfrac{\dfrac{(2x)^{2n+2}}{x^{n+1}}}{\dfrac{(2x)^{2n}}{x^{n}}}\right| = \lim_{x\to\infty}\left|\dfrac{4x^2}{x}\right| = |4x|$ $\quad |4x|<1 \quad -\dfrac{1}{4}<x<\dfrac{1}{4}$ Check endpoints.

$\displaystyle\sum_{n=1}^{\infty}\frac{\left(2\left(-\frac{1}{4}\right)\right)^{2n}}{\left(-\frac{1}{4}\right)^{n}} = \sum_{n=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^{2n}}{\left(-\frac{1}{4}\right)^{n}} = \sum_{n=1}^{\infty}(-1)^n$ divergent $\displaystyle\sum_{n=1}^{\infty}\frac{\left(2\left(\frac{1}{4}\right)\right)^{2n}}{\left(\frac{1}{4}\right)^{n}} = \sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\right)^{2n}}{\left(\frac{1}{4}\right)^{n}} = \sum_{n=1}^{\infty}(1)^n$ divergent.

30) D: The third term of $P'(x)$ will be the derivative of the 4$^{th}$ term or $\cos\left(\frac{\pi}{4}\right)\dfrac{\left(x-\frac{\pi}{4}\right)^4}{4!}$.

$$\frac{d}{dx}\left(\cos\left(\frac{\pi}{4}\right)\frac{\left(x-\frac{\pi}{4}\right)^4}{4!}\right) = \frac{d}{dx}\left[\frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^4}{24}\right] = \frac{\sqrt{2}}{12}\left(x-\frac{\pi}{4}\right)^3 \qquad \frac{\sqrt{2}}{12}$$