Solutions

1. Triangle area: $\sqrt{14(4)(2)(8)} = 8\sqrt{14} \Rightarrow A = 8,\ B = 14$

   $r = \frac{A}{s} \to \frac{\frac{36\sqrt{3}}{4}}{\frac{18}{2}} = \sqrt{3}$; circle area $3\pi \Rightarrow C = 3$

   $2x^3 + 11x^2 - 7x - 6 \ge 0 \to (x+6)(2x+1)(x-1) \ge 0 \to \left[-6, -\frac{1}{2}\right] \cup [1, \infty) \Rightarrow D = -6$

   $\left[\log_3 \frac{24}{11}\right] \to [\log_3 24 - \log_3 11] \to [\log_3(8 \bullet 3) - \log_3 11] \to [\log_3 8 + \log_3 3 - \log_3 11] \to$
   $[1.9 + 1 - 2.2] \to [0.7] = 0 \Rightarrow E = 0$

   $8 + 14 + 3 - 6 + 0 = 19$

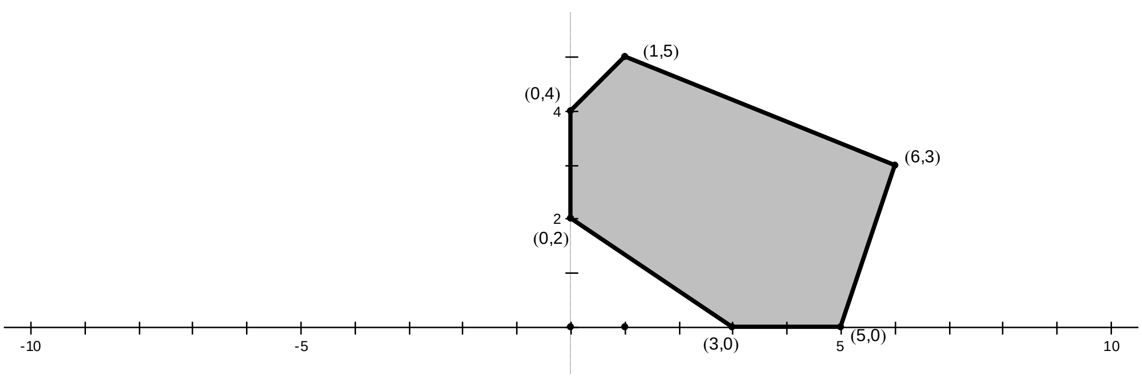
2. $10^{2009} - 10^{2008}$ gives 1000 9's, so the sum is $9000 \Rightarrow A$

   $3x + 2 = 1$ where $x = \frac{1}{3}$, so $\left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) - 3 = -\frac{23}{9} \Rightarrow B$

   Original determinant is 783: $10(0+84) - 1(-24-24) + 5(-21+0)$. Since the matrix is of order 3, the determinant is multiplied by 2^3: $783 \times 8 = 6264 \Rightarrow C$

   $A + B + C = 9000 + 24601 - \frac{23}{9} = \frac{24601}{9} \Rightarrow 24601.$

3. Max = 51, Min = 14, Diff = 37.



4. $\frac{x^2 + 2x - 8}{4x^3 - 32} > 0 \to \frac{(x+4)(x-2)}{4(x-2)(x^2 + 2x + 4)} > 0$. The inequality is undefined at $x = 2$.

   The solution set is $(-4, 2) \cup (2, \infty)$. The integers less than 5 are $-3, -2, -1, 0, 1, 3, 4$, the sum of which is $2 \Rightarrow A$.

   The letters that are not used are HHW, so $\frac{3!}{2!} = 3 \Rightarrow B$

   Using the given information, we find $\cos\alpha = -\frac{4}{5}$ and $\cos\beta = -\frac{12}{13}$.

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta=\left(\frac{3}{5}\right)\left(-\frac{12}{13}\right)+\left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)=-\frac{16}{65}\Rightarrow C$$

$$ABC=(2)(3)\left(-\frac{16}{65}\right)=-\frac{96}{65}.$$

5. The first four partial sums add to $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}$. This shows that the total sum will be (one less than right-most denominator)/(right-most denominator). Here, it will be $\frac{10!-1}{10!}=\frac{8628799}{8628800}\Rightarrow \frac{A}{B}$.

   Let a, b be the integers with $a>b$. Thus, $\frac{a+b}{2}-50=\sqrt{ab}\rightarrow a-2\sqrt{ab}+b=100\rightarrow$ $\left(\sqrt{a}-\sqrt{b}\right)^2=100\rightarrow\sqrt{a}-\sqrt{b}=\pm10$. Since $a>b$, the difference must be $10\Rightarrow C$.

   $\frac{1}{x}=\frac{\frac{1}{9}+\frac{3}{7}}{2}=\frac{17}{63}\rightarrow x=\frac{63}{17}$.

   $B-A+C+D+E=1+10+80=91.$

6. The sum of the reciprocals of the divisors is (sum of divisors)/(the number).
   $2010=2^1\cdot3^1\cdot5^1\cdot67^1$, so the sum of the positive divisors is $(1+2)(1+3)(1+5)(1+67)$, which is 4896. $\frac{4896}{2010}=\frac{816}{335}\Rightarrow A$.

   There are ${}_6C_3=20$ ways to assign the seats to the boys. Some of these 20 ways will place all three boys together and thus cannot be used: BBBGGG, GBBBGG, GGBBBG, GGGBBB. There are also ways to sit all three girls together: GGGBBB, BGGGBB, BBGGGB, BBBGGG. Notice that two of these are used twice, so a total of six of these cannot be used. Therefore, 14 of these 20 ways work. There are 3! ways to place the boys in these arrangements and also 3! to place the girls. Overall, there are $14\times3!\times3!=504$ ways to place the children accordingly. $504\Rightarrow B$

   $(8+1+6)+(3+3+5)+(5+0+4)=35$.

7. Maximum value of $y=-41\cos3x+8$ is (41 + 8) = 49 and the period is $\frac{360}{3}=120$. Sum is $169\Rightarrow A$.

   Maximum value of $y=3\cos2x-4\sin2x$ is 5 and period is 180. Sum is $185\Rightarrow B$.

   Area is $\frac{1}{2}(13)(15)\sin135=\frac{195}{2}\left(\frac{\sqrt{2}}{2}\right)=\frac{195\sqrt{2}}{4}\Rightarrow C$.

   $\overline{OJ}=\left\langle-4\sqrt{3}, 4\right\rangle$ and $\overline{OL}=\left\langle-4\sqrt{2}, -4\sqrt{2}\right\rangle$. $(JL)^2=\left(-4\sqrt{3}+4\sqrt{2}\right)^2+\left(4+4\sqrt{2}\right)^2=$ $160-32\sqrt{6}+32\sqrt{2}$.

   $$B-A+C+D=176-32\sqrt{6}+\frac{323\sqrt{2}}{4}=\frac{704-128\sqrt{6}+323\sqrt{2}}{4}.$$

8. $\begin{cases} x - y = 9 \\ \dfrac{x}{y} = 9 \end{cases} \rightarrow y = \dfrac{x}{9} \rightarrow x - \dfrac{x}{9} = 9 \rightarrow x = \dfrac{81}{8}$. The most common digit is $8 \Rightarrow A$.

$B = \dfrac{(200)(201)}{2} = 20100$

$C = 6 + 28 + 496 = 530$

$205 = 5 \cdot 41,\ 408 = 2^3 \cdot 3 \cdot 17,\ 45 = 3^2 \cdot 5 \rightarrow 2^3 \cdot 3^2 \cdot 5 \cdot 17 \cdot 41 \rightarrow (3+1)(2+1)(1+1)^3 \rightarrow 96$ factors.

Since positive/negative wasn't specified, there are 192 total factors $\Rightarrow D$.

$A + B + C + D = 8 + 20100 + 530 + 192 = 20830$.

9. $\sin 5x + \sin x = 0 \rightarrow 2\sin 3x \cos 2x = 0 \rightarrow \sin 3x \cos 2x = 0$.

For $\sin 3x = 0,\ x = 0, 60, 120, 180, 240$. For $\cos 2x = 0,\ x = 45, 135, 225, 315$. Arranged numerically we have: $0, 45, 60, 120, 135, 180, 225, 240, 300, 315$. Since we have an even number of solutions, the median is the mean of the middle two entries:

$\dfrac{135 + 180}{2} = 157.5 \Rightarrow A$.

$\dfrac{1 + \cos\theta}{\sin\theta} = -1 \rightarrow \cot\dfrac{1}{2}\theta = -1 \rightarrow \dfrac{1}{2}\theta = 135 + 180n \rightarrow \theta = 270 \Rightarrow -90 = B$

$\tan\theta - \tan 10° = 1 + \tan\theta\tan 10° \rightarrow \dfrac{\tan\theta - \tan 10°}{1 + \tan\theta\tan 10°} = 1 \rightarrow \tan(\theta - 10°) = 1 \rightarrow$

$\theta - 10° = 45 + 180n° \Rightarrow \theta = \{55°, -125°\} \Rightarrow C$

$2\sin^2 x + \cos x \geq 1 \rightarrow 2(1 - \cos^2 x) + \cos x \geq 1 \rightarrow 2\cos^2 x - \cos x - 1 \leq 0 \rightarrow$

$(2\cos x + 1)(\cos x - 1) \leq 0 \rightarrow \cos x = -\dfrac{1}{2},\ \cos x = 1 \rightarrow$

$[0, 120] \cup [240, 360] \Rightarrow [D, E] \cup [F, G]$

$A + B + C + E = 315 + (-90) + (55 + (-125)) + 120 = 275$.

10. $4y^2 - 9x^2 + 16y + 18x = 29 \rightarrow \dfrac{(y+2)^2}{9} - \dfrac{(x-1)^2}{4} = 1,\ a^2 = 9, b^2 = 4, c^2 = 13$

$A = e = \dfrac{c}{a} = \dfrac{\sqrt{13}}{3}$

$B = 2c = 2\sqrt{13}$

Vertices: $(1,1), (1,-5)$, so $C = 6$

D is distance between $\left(1, -2 + \sqrt{13}\right)$ and $3x - 2y - 7 = 0$: $\dfrac{\left|3(1) - 2\left(-2 + \sqrt{13}\right) - 7\right|}{\sqrt{3^2 + (-2)^2}} = 2$

$E = C = 2a = 6$

$F = \dfrac{2b^2}{a} = \dfrac{8}{3}$.

$$\frac{ABC}{DEF} = \frac{\left(\frac{\sqrt{13}}{3}\right)\left(2\sqrt{13}\right)(6)}{(2)(6)\left(\frac{8}{3}\right)} = \frac{13}{8}$$

$A = 2(1-2i)^3 - 3(1-2i)^2 + 5(1-2i) - 1 = -9 + 6i$

$$z = \frac{3-2i}{2+i} \cdot \frac{2-i}{2-i} = \frac{4}{5} - \frac{7}{5}i \rightarrow \bar{z} = \frac{4}{5} + \frac{7}{5}i \Rightarrow B$$

$z_1 = 0, z_2 = z_1^2 + i = i, z_3 = z_2^2 + i = -1 + i, z_4 = z_3^2 + i = (-1+i)^2 + i = -i$. For z_n with even n, $n \geq 3$, $z_n = -i$; for odd n, $z_n = -1 + i$. $z_{2008}^2 + z_{2009}^2 = (-i)^2 + (-1+i)^2 = -1 - 2i$.

$A + 5B + C = -6 + 11i$.