2009 Mu School Bowl

1. Given $f(x) = 2x^3 - 9x^2 + 12x + 1$

   $\boldsymbol{A}$ = the slope of the line tangent to $f(x)$ at $x = -3$.

   $\boldsymbol{B}$ = the y – intercept of the line tangent to $f(x)$ at $x = 2$

   $\boldsymbol{C}$ = y – value of the point of inflection of the graph of $f(x)$

   $\boldsymbol{D}$ = the value of $\int_0^3 f(x)dx$.

   Find: $\sqrt{\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} + \boldsymbol{D}}$

2. $\boldsymbol{A} = det\left(\begin{bmatrix} -2 & 5 & 1 \\ 3 & 7 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & 0 \\ 1 & 5 \end{bmatrix}\right)$

   $\boldsymbol{B}$ = the value of $f(f(f(2)))$ if $f(x) = 3x^2 - 2x - 1$ when x is even and $f(x) = x^2 - 5x + 1$ when x is odd.

   $\boldsymbol{C}$ = the area enclosed by $y = |x - 1| + |x + 2|$ and $y = |x - 3| - |x + 2|$.

   D = $Tan\left(Cos^{-1}\left(\frac{6}{7}\right)\right)$.

   Find: $\boldsymbol{A} - \boldsymbol{B} + \boldsymbol{C} + 13\boldsymbol{D}^2$.

3. $\boldsymbol{A} = \sum_2^{100} \frac{3}{x(x-1)}$

   $\boldsymbol{B}$ = the sum of the real zeros of $f(x) = x^4 - 3x^3 - 5x^2 + 23x + 66$.

   C = the length of the graph $f(x) = ln\, x - \frac{1}{8}x^2$ for $1 \le x \le 2$.

   D = the derivative at the point (-1, 2), given $x^2 - xy^2 + y^3 = 13$.

   Find: $100\boldsymbol{A} - 4(\boldsymbol{B} + \boldsymbol{C} + \boldsymbol{D})$.

4. ***A*** = the sum of x and y in the solution to: $3(x + y)\boldsymbol{i} - 3\boldsymbol{i} + 2x = 5 + 7\boldsymbol{i}$.

   B = the value of k in $\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}} = \left(\frac{a}{b}\right)^k$ for all $a, b \geq 0$

   C = the sum of the positive value(s) of x if $4^{\log_2 x} + x^2 = 8$

   D = length of the altitude to the longest side of a triangle with side lengths of 13, 37, and 40.

   Find **ABCD**

5. Given : $f(x) = 5x^5 - 4x^3 - 2x - 4$.

   A = the x – value of the positive inflection point.

   B = the number of real zeros + the number of local extrema.

   C = the area between $f(x)$ and $y = -x - 4$ in the interval [0, 1].

   $D = \lim_{x \to \frac{\sqrt{6}}{5}} \frac{y''}{5x - \sqrt{6}}$.

   Find: $\boldsymbol{A}^2 + \boldsymbol{BC} - \boldsymbol{D}$

6. ***A*** = Area of the ellipse: $9x^2 + 4y^2 - 18x + 16y = 11$

   B = the number of integer pairs that satisfy the conditions: $y > |x - 1| + |x - 5|$ and $y < 6$.

   C = is the eccentricity of the conic with an equation of: $4x^2 - y^2 + 24x - 10y - 15 = 0$.

   D = the length of the transverse axis of the hyperbola: $x^2 - 4y^2 + 4x + 32y - 96 = 0$

   Find: ***AB*** + ***CD***

7. A = the number of positive integer pairs (x, y) that are solutions to $5x + 3y = 87$.

B = the coefficient of the 6th term in the expansion of $(2x - y)^8$.

C = the number of positive ordered pairs satisfying $x = \frac{6-x}{y^2 - x}$.

D = the simplified form of: $\left(\sqrt[3]{\sqrt{75 - \sqrt{12}}}\right)^{-2}$.

Find: $\frac{\boldsymbol{B}}{\boldsymbol{ACD}}$

8. $A = a + b + c$, *given:* $\sum_2^{2008} \log_2 \frac{n}{n+1} = a + b\, log_2\, c$.

B = the sum of the solution(s) to: $ln(3x - 1) + ln(x + 3) = 2\, ln\, 5$.

C = the value of $log_n \frac{(a\sqrt{b})^2}{b^2 c}$, if $log_n a = 2$, $log_n b = 3$, $log_n c = 5$.

$D = k$ if $\log_y x + \log_{y^2} x = 6$, then $x = y^k$.

Find the sum of the digits in the simplified form of $\boldsymbol{2A}/\boldsymbol{B} + \boldsymbol{CD}$.

9. $\boldsymbol{A} = f\left(\frac{1}{2}\right)$ if $f'(x) = x\sqrt{1 - x^2}$ and $f(0) = 3$.

$\boldsymbol{B} = lim_{x\to\infty} \frac{\ln(x^2 + 1)}{\ln x}$.

$\boldsymbol{C}$ = is the derivative of y with respect to x at $\left(3\pi/4, -2\right)$ of $y^2 sin\, 2x = 2y$.

$\boldsymbol{D}$ = the area A of the region between the graph of $f(x) = 3x^2 + 4$ and the x – axis in the interval [-1, 1].

Find: $(\boldsymbol{A} + \boldsymbol{C})(\boldsymbol{B} + \boldsymbol{D})$

10. $\boldsymbol{A}$ = the smaller value of x which satisfies $\log_9 x + \frac{1}{\log_9 x} = 5/2$.

$\boldsymbol{B}$ = the smallest of 3 integers that form a geometric progression if their sum is 21 and the sum of their reciprocals is $^7/_{12}$.

$\boldsymbol{C}$ = the area of rectangle R_2 if it's diagonal is 15 and is similar to rectangle R_1 with one side 2 and area 12.

$\boldsymbol{D} = a + b$ if $\sqrt{20 + \sqrt{384}} = \sqrt{a} + \sqrt{b}$ and $a < b$.

Find: $\dfrac{\boldsymbol{C}^2}{\boldsymbol{A}^{\boldsymbol{B}}\boldsymbol{D}}$