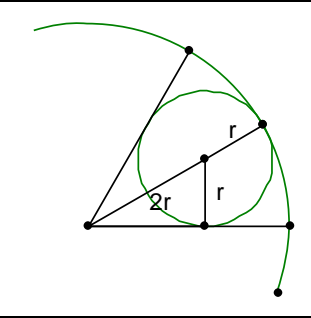
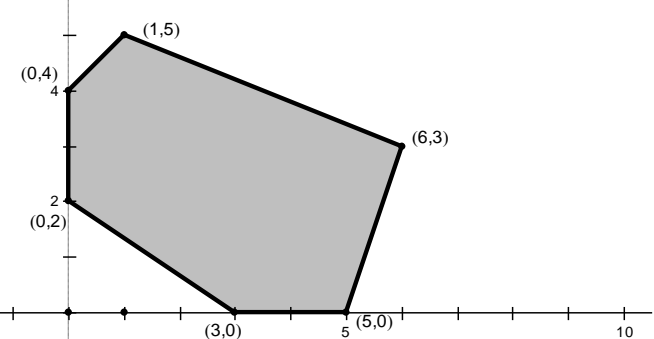
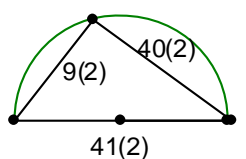
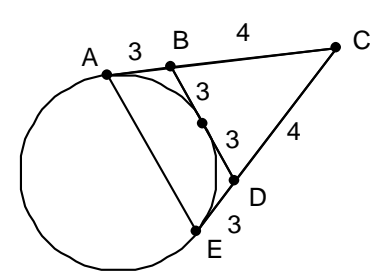
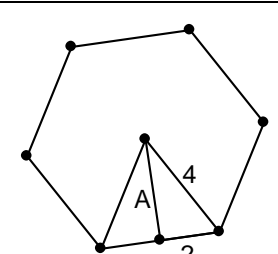
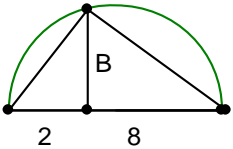
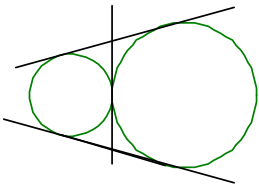


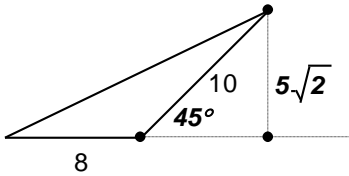
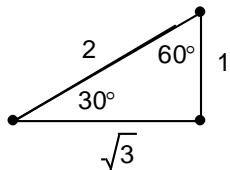
2009 Theta School Bowl
Solutions

1.	A = 8 B = 14	Triangle with sides 10, 12, 6: $Area = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{14(4)(2)(8)} = 8\sqrt{14}$	
	C = 3	$A = rs; r = \frac{A}{s}; r = \frac{36\sqrt{3}/4}{18/2} = \sqrt{3}; Area\ of\ triangle = 3\pi$	
	D = -6	$2x^3 + 11x^2 - 7x - 6 \geq 0 \rightarrow (x+6)(2x+1)(x-1) \geq 0 \Rightarrow \left[-6, -\frac{1}{2}\right] \cup [1, \infty)$	
	E = 0	$\left[\log_3 \frac{24}{11}\right] \rightarrow [\log_3 24 - \log_3 11] \rightarrow [\log_3(8 \cdot 3) - \log_3 11] \rightarrow [\log_3 8 + \log_3 3 - \log_3 11] \rightarrow [1.9 + 1 - 2.2] \rightarrow [0.7] = 0$	
	Ans. 19	$8 + 14 + 3 - 6 + 0 = 19$	
2.	A = $\frac{5}{3}$	$3r = 5; r = \frac{5}{3}$	
	B = 420	$V = \frac{kC^2}{2R} \rightarrow 768 = \frac{64k}{10} \rightarrow k = 120; V = \frac{120(49)}{2(7)} = 420$	
	C = $\frac{3}{25}$	$\frac{4}{50} + \frac{2}{50} = \frac{6}{50} = \frac{3}{25}$	
	Ans. 84	$\left(\frac{5}{3}\right)(420)\left(\frac{3}{25}\right) = 84$	
3.	A = $\frac{60}{13}$	$alt = \frac{l_1 l_2}{h} = \frac{(5)(12)}{13} = \frac{60}{13}$	
	B = 1	$A = rs; r = \frac{A}{s}; r = \frac{.5(3)(4)}{6} = \frac{6}{6} = 1$	
	C = 6264	Original determinant is 783: $10(0+84) - 1(-24-24) + 5(-21+0)$. Since the matrix is of order 3, the determinant is multiplied by 2^3 . $8 \times 783 = 6264$.	
	D = $6\sqrt{2}$	$3(x^2 + 4x + 4) + (y^2 + 6y + 9) = 6 + 12 + 9 \rightarrow 3(x+2)^2 + (y+3)^2 = 27 \rightarrow \frac{(x+2)^2}{9} + \frac{(y+3)^2}{27} = 1$. $a^2 - b^2 = c^2 \rightarrow 18 = c^2 \rightarrow c = 3\sqrt{2}$. Distance = $2c = 6\sqrt{2}$.	
	Ans. 6	Most commonly used digit in A, B, C, D is 6.	

<p>4.</p> <p>Max = 51, Min = 14 Diff = 37</p>		
<p>Ans. 37 At (0,2): $z = 5(0)+7(2) = \mathbf{14}$; At (0,4): $z = 5(0)+7(4) = 28$; At (1,5): $z = 5(1)+7(5) = 40$; At (6,3): $z = 5(6)+7(3) = \mathbf{51}$; At (5,0): $z = 5(5)+7(0) = 25$; At (3,0): $z = 5(3)+7(0) = 15$</p>		
<p>5.</p> <p>$A = \frac{1681\pi}{2}$</p>		<p>$Area = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(41)^2 = \frac{1681\pi}{2}$</p>
<p>$X = \frac{49}{2}$</p>		<p>Since triangles BCD and ACE are similar triangles with ratio of similitude is $\frac{4}{7}$ that is also the ratio of the perimeters. $\frac{4}{7} = \frac{14}{X}$; $X = \frac{(7)(14)}{4} = \frac{49}{2}$</p>
<p>F = 2</p>	<p>$\frac{x^2 + 2x - 8}{4x^3 - 32} > 0 \rightarrow \frac{(x+4)(x-2)}{4(x-2)(x^2 + 2x + 4)} > 0$. The inequality is undefined at $x = 2$. The solution set is $(-4, 2) \cup (2, \infty)$. The integers less than 5 are $-3, -2, -1, 0, 1, 3,$ and 4, the sum of which is 2.</p>	
<p>G = 1680</p>	<p>THETASCHOOLBOWL will become HHOOLOWL. $\frac{8!}{2!3!2!} = 1680$.</p>	
<p>Ans. 3413+1681π</p>	<p>$2\left(\frac{1681\pi}{2} + \frac{49}{2} + \frac{4}{2} + \frac{3360}{2}\right) = 3413 + 1681\pi$</p>	
<p>6.</p> <p>$A = 2\sqrt{3}$</p>		<p>$A = 2\sqrt{3}$</p>
<p>B = 18</p>	<p>$3x-12 + 2 \leq x+5 - 3 \rightarrow 3 x-4 + 5 \leq x+5$. Graphing each side of the inequality, we have $y = -x-5$, $y = x+5$, $y = 3x-7$, $y = -3x+17$. $y = 3x-7$ and $y = x+5$ intersect where $x = 6$. $y = x+5$ and $y = -3x+17$ intersect at $x = 3$. The interval $[3, 6]$ is the interval on which the left side of the inequality has values less than or equal to the right side. The sum</p>	

		of these numbers is 18.
	$C = \frac{22}{5}$	$\begin{cases} \log xy^3 = 4 \\ \log x^2y = 9 \end{cases} \rightarrow \begin{cases} \log x + 3\log y = 4(1) \\ 2\log x + \log y = 9(-3) \end{cases} \rightarrow \begin{cases} \log x + 3\log y = 4 \\ -6\log x - 3\log y = -27 \end{cases}$ $-5\log x = -23 \rightarrow \log x = \frac{23}{5}. \quad \log y = 9 - 2\left(\frac{23}{5}\right) = \frac{45 - 46}{5} = -\frac{1}{5}.$ $\log xy = \log x + \log y = \frac{22}{5}.$
	Ans. $20\sqrt{3} + 224$	$10\left(2\sqrt{3} + 18 + \frac{22}{5}\right) = 20\sqrt{3} + 180 + 44 = 224 + 20\sqrt{3}.$
7.	A = 133	Space diagonal rectangular solid: $\sqrt{A} = \sqrt{l^2 + w^2 + h^2} = \sqrt{6^2 + 4^2 + 9^2} = \sqrt{36 + 16 + 81} = \sqrt{133}$
	B = 4	 <p>Since the triangle is inscribed in a semicircle it is a right triangle and the post is the altitude to the hypotenuse. The post B is the geometric mean between 2 and 8. $B^2 = (8)(2) = 16; \quad B = 4$</p>
	C = 5	$\begin{bmatrix} 3 & \frac{1}{7} \\ -\frac{1}{2} & \frac{3}{14} \end{bmatrix}. \quad \det = \frac{9}{14} - \left(-\frac{1}{14}\right) = \frac{5}{7}. \quad \frac{1}{\frac{5}{7}} \begin{bmatrix} \frac{3}{14} & -\frac{1}{7} \\ \frac{1}{2} & 3 \end{bmatrix} \rightarrow \frac{7}{5} \begin{bmatrix} \frac{3}{14} & -\frac{1}{7} \\ \frac{1}{2} & 3 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{3}{10} & -\frac{1}{5} \\ \frac{7}{10} & \frac{21}{5} \end{bmatrix}.$
	D = 504	There are ${}_6C_3 = 20$ ways to assign the seats to the boys. Some of these 20 ways will place all three boys together and thus cannot be used: BBBGGG, GBBBGG, GGBBBG, GGGBBB. There are also ways to sit all three girls together: GGGBBB, BGGGBB, BBGGGB, BBBGGG. Notice that two of these are used twice, so a total of six of these cannot be used. Therefore, 14 of these 20 ways work. There are 3! ways to place the boys in these arrangements and also 3! ways to place the girls. Overall, there are $14 \times 3! \times 3! = 504$ ways to place the children accordingly.
	Ans. 138	The only prime answers are 133 and 5, so the sum is 138.
8.	A = 3	 <p>There are two common external tangents and one common internal tangent for a pair of externally tangent circles.</p>
	B = -1	$\frac{x}{x-2} + \frac{1}{x-4} = \frac{2}{x^2 - 6x + 8} \rightarrow x(x-4) + 1(x-2) = 2 \rightarrow x^2 - 3x - 4 = 0. \quad x = -1$ only as $x = 4$ is an excluded value.
	C = $-\frac{3}{2}$	Since we are taking the roots two at a time, we can use $\frac{(-1)^2 a_{n-2}}{a_n} \Rightarrow \frac{1(-6)}{4} = -\frac{3}{2}.$
	Ans. $-\frac{8}{27}$	$\left(-\frac{3}{2}\right)^{-3} = -\frac{8}{27}.$

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<p>9.</p> <p>$\frac{T\sqrt{U}}{V} = \frac{2\sqrt{13}}{13}$</p> <p>$T = 2$ $U = 13$ $V = 13$</p>		<p>Distance from point (x_1, y_1) to line $ax + by + c = 0$ is $\frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$.</p> <p>$y = \frac{2}{3}x - 1$ becomes $2x - 3y - 3 = 0$ and distance = $\frac{ 2(5) + (-3)(3) - 3 }{\sqrt{2^2 + (-3)^2}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$</p>	
<p>$W\sqrt{X} = 20\sqrt{2}$</p> <p>$W = 20$ $X = 2$</p>			<p>$Area = \frac{1}{2}bh = \frac{1}{2}(8)(5\sqrt{2}) = 20\sqrt{2}$</p>
<p>$C = 76$</p>		<p>$\log_{0.2}(x-1) + \log_{25} 9 = -2 \rightarrow \log_{5^{-1}}(x-1) + \log_{5^2} 3^2 = -2 \rightarrow -\log_5(x-1) + \log_5 3 = -2.$</p> <p>$\log_5 \frac{3}{x-1} = -2 \rightarrow \frac{3}{x-1} = \frac{1}{25}. \quad x = 76.$</p>	
<p>$D = 11$</p>		<p>There is a faster way to work this problem, but for kicks let's do it the "long" way. $9 = 3^2$, so we need to find out how many 3's are in 50!. The multiples of 3 that are contained in 50! are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, and 48, for a total of 22 3's. If there are 22 3's, then there are 11 9's.</p>	
<p>Ans. $104\sqrt{1045}$</p>		<p>$\sqrt{2 \times 13 \times 13 \times 20 \times 2 \times 76 \times 11} \rightarrow \sqrt{13 \times 13 \times 4 \times 5 \times 4 \times 4 \times 19 \times 11} = 104\sqrt{1045}$</p>	
<p>10.</p> <p>$A = \sqrt{3}$</p>			<p>Tangent of an acute angle of a right triangle is equal to the length of the opposite leg over the adjacent leg.</p> <p>$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$</p>
<p>$B = -\frac{81}{5}$</p>		<p>Using expansion of minors, we have</p> <p>$x(0-10) - 3(-3+50) + 2(-2-0) = 17 \rightarrow -10x = 17 + 3(47) + 4 \rightarrow x = -\frac{162}{10} = -\frac{81}{5}.$</p>	
<p>$C = \frac{20}{9}$</p>		<p>$y = \frac{4x^2 - 8x + 2}{6x^2 - 22x + 20} = \frac{2(2x^2 - 4x + 1)}{2(3x - 5)(x - 2)}$. VA at $x = \frac{5}{3}, x = 2$. HA at $y = \frac{2}{3}$. $\left(\frac{5}{3}\right)(2)\left(\frac{2}{3}\right) = \frac{20}{9}$</p>	
<p>Ans. $-36\sqrt{3}$</p>		<p>$(\sqrt{3})\left(-\frac{81}{5}\right)\left(\frac{20}{9}\right) = -36\sqrt{3}.$</p>	
<p>11.</p> <p>$A = 156$</p>		<p>$ext \square = \frac{360}{15} = 24 \quad A = int \square = 180 - ext \square = 180 - 24 = 156$</p>	
<p>$B = 1$</p>		<p>$p \quad q \quad \square p \quad \square q \quad (\square p \vee q) \quad (p \wedge \square q) \quad (\square p \vee q) \rightarrow (p \wedge \square q)$</p> <p>$T \quad T \quad F \quad F \quad T \quad F \quad F$</p> <p>$T \quad F \quad F \quad T \quad F \quad T \quad T$</p> <p>$F \quad T \quad T \quad F \quad T \quad F \quad F$</p> <p>$F \quad F \quad T \quad T \quad T \quad F \quad F$</p>	
<p>$C = 144$</p>			<p>$\left[(\log_2 x)(\log_3 y)(\log_x 8)(\log_y 81)\right]^2 = \left[\left(\frac{\log x}{\log 2}\right)\left(\frac{\log y}{\log 3}\right)\left(\frac{\log 8}{\log x}\right)\left(\frac{\log 81}{\log y}\right)\right]^2 =$</p>

		$\left[\left(\frac{3\log 2}{\log 2}\right)\left(\frac{4\log 3}{\log 3}\right)\right]^2 = [12]^2 = 144$
	D = 0	Synthetically divide by 13 to find the remainder which is P(13). $\begin{array}{r} 13 \overline{) 1 \ -12 \ -26 \ 169} \\ \underline{0 \ 13 \ 13 \ -169} \\ 1 \ 1 \ -13 \ 0 \end{array}$
	Ans. 300	$A + C + D = 156 + 144 + 0 = 300$
12.	A = 2π	This is the length of a 45° arc of a great circle of a sphere whose diameter is 8. Distance traveled = $\frac{90^\circ}{360^\circ} \pi(8) = 2\pi$
	B = -10	$\frac{1}{2} \begin{vmatrix} x & 7 & 1 \\ -5 & x & 1 \\ x & 1 & 1 \end{vmatrix} = \pm 10 \rightarrow \begin{vmatrix} x & 7 & 1 \\ -5 & x & 1 \\ x & 1 & 1 \end{vmatrix} = \pm 20$. $x(x-1) - 7(-5-x) + 1(-5-x^2) = \pm 20$. $x^2 - x + 35 + 7x - 5 - x^2 = \pm 20 \rightarrow 6x + 30 = \pm 20$. $x = -\frac{25}{3}$ or $x = -\frac{5}{3}$. Sum = -10.
	C = 40	$x^2 + 6x + 25y^2 + 100y + 9 = 0 \rightarrow x^2 + 6x + 9 + 25(y^2 + 4y + 4) = -9 + 9 + 100$. $\frac{(x+3)^2}{100} + \frac{(y+2)^2}{4} = 1$. Endpoints will be (-3, 0), (-3, -4), (-13, -2), and (7, -2). $\frac{1}{2} \begin{vmatrix} -3 & 0 \\ 7 & -2 \\ -3 & -4 \\ -13 & -2 \\ -3 & 0 \end{vmatrix} \rightarrow \frac{1}{2} [(6 - 28 + 6 + 0) - (0 + 6 + 52 + 6)] \rightarrow \frac{1}{2} (-16 - 64) = 40$.
	Ans. 30π	$LA = \pi r l = \frac{1}{2} Cl = \frac{1}{2} (2\pi)(30) = 30\pi$.