

**E is none of these**

- Find the sum of the 3 highest primes that divides  $5^{12} - 4^{12}$ ?  
a) 139                      b) 205                      c) 623                      d) 823
- What is the smallest positive number that has exactly 10 positive factors?  
a) 12                      b) 24                      c) 36                      d) 48
- What is the highest prime number that always divides  $aaabbb$  where  $a$  and  $b$  are positive integers?  
a) 37                      b) 29                      c) 13                      d) 3
- The polynomials  $x^3 + ax^2 + 9x + 6$  and  $x^3 + bx^2 + 6x + 3$  have a common quadratic factor over the set of polynomials with integral coefficients. Find the ordered pair of integrals  $(a, b)$ .  
a) (2, 3)                      b) (5, 4)                      c) (3, 4)                      d) (5, 2)
- What is the remainder when  $9^{83} + 5^{32}$  is divided by 6?  
a) 3                      b) 2                      c) 5                      d) 4
- If  $154_b + 345_b = 268_{10}$  find the value of  $b$ .  
a) 6                      b) 7                      c) 9                      d) 12
- Find the number of distinct positive integral divisors of  $(30)^4$  excluding 1 and 30.  
a) 64                      b) 100                      c) 123                      d) 148
- Define  $n_{a!}$  for  $n$  and  $a$  positive to be:  $n_{a!} = n(n - a)(n - 2a)(n - 3a)\dots(n - ka)$ , where  $k$  is the greatest integer for which  $n > ka$ . Find the quotient  $\frac{72_{8!}}{18_{2!}}$ .  
a)  $2^{17}$                       b)  $4^6$                       c)  $2^{15}$                       d)  $4^9$
- The number 32639 is the product of two primes, one of which is roughly twice as large as the other. Find the sum of these two primes.  
a) 384                      b) 381                      c) 378                      d) 356

10. Find the number of positive integers less than 800 that is not divisible by 5 or 7.
- a) 246                      b) 268                      c) 553                      d) 576
11. Given:  $S_n = \{S_1, S_2, S_3, \dots, S_k\}$ , where  $S_1 = 2$ ,  $S_2 = 4 + 6$ ,  $S_3 = 8 + 10 + 12, \dots$   
Continuing in the same pattern where each new sum is one term more than the preceding one, with the 1<sup>st</sup> term of the new sum is two more than the last term of the last sum, find the sum of  $S_{20}$ .
- a) 8020                      b) 9120                      c) 10200                      d) 12020
12. Find the units digit in the expression  $2007^{2009}$ .
- a) 9                              b) 3                              c) 1                              d) 7
13. For every positive integer  $n$ ,  $7^{2n+1} + 1$  is divisible by the positive integer  $k$ . Find the largest value of  $k$ .
- a) 2                              b) 4                              c) 8                              d) 16
14. The three digit number  $2a3$  is added to 326 to give the 3-digit number  $5b6$ . If  $5b9$  is divisible by 9, find  $a + b$ .
- a) 9                              b) 6                              c) 5                              d) 4
15. In the following equation each of the letters represents a uniquely different digit in base ten.  $BA \cdot MA = QQQ$ . Find the sum of  $A + M + Q + Y$ .
- a) 21                              b) 20                              c) 19                              d) 18
16. The number  $(2^{48} - 1)$  is exactly divisible by two numbers between 60 and 70. Find the sum of the two numbers.
- a) 125                              b) 126                              c) 127                              d) 128
17. The number  $N$  is represented by the base  $q$  numeral 1441. When divided by eleven,  $N$  leaves a remainder of 1. If  $1 < q \leq 10$ , find  $q$ .
- a) 3                              b) 5                              c) 7                              d) 9

18.  $AB$  and  $CA$  are decimal numerals, and  $A, B, C$  are distinct digits. If 4 times  $AB$  equals  $CA$ , find the value of  $C$  in the ordered triple  $(A,B,C)$
- a) 2                      b) 3                      c) 6                      d) 9
19. A sequence of digits has the property that each pair of successive digits, taken in the order written, forms a decimal numeral representing a multiple of either 17 or 23. If the first digit is 9 and the sequence is finite, what is its last (possible) digit?
- a) 1                      b) 3                      c) 5                      d) 7
20. Find the greatest common factor of all numbers of the form  $2^n \cdot 3^n - 1$ , where  $n$  is an integer greater than 1.
- a) 5                      b) 11                      c) 17                      d) 43
21. Find the largest number  $n$  for which  $5^n$  is a factor of  $309!$
- a) 75                      b) 63                      c) 61                      d) 47
22. Find the number of ordered triples  $(A, B, C)$  where  $A, B$  and  $C$  are positive integers,  $A$  is a factor of  $B$ ,  $A$  is a factor of  $C$ ,  $A < B < C$ , and  $A+B+C=30$ .
- a) 4                      b) 8                      c) 9                      d) 11
23. What is the remainder of the division of  $14414 \cdot 14416 \cdot 14418$  by 14?
- a) 4                      b) 7                      c) 8                      d) 13
24.  $MNP$  and  $PNM$  are, respectively, the base 9 and base 7 numerals for the same positive integer  $ABC$  in base ten. Find  $M + N + P$ .
- a) 8                      b) 7                      c) 6                      d) 5
25. In how many zeros does  $2009!$  end?
- a) 401                      b) 441                      c) 457                      d) 480
26. The infinite sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23... is obtained by writing the positive integers in order.

2009 Number Theory (Open)

What is the 2009<sup>th</sup> digit in this sequence?

- a) 4                                      b) 3                                      c) 2                                      d) 1

27. 100 is divided into two parts such that one part is divisible by 7 and the other part is divisible by 11. Find the product of the 2 parts.

- a) 2448                                      b) 2464                                      c) 2664                                      d) 2848

28. How many of the numbers denoted by  $3^1, 3^2, 3^3, \dots, 3^{99}, 3^{2009}$ , written in base 10 notation end in 1?

- a) 325                                      b) 405                                      c) 551                                      d) 573

29. If  $A$  and  $B$  are digits and  $30AB5$  can be expressed as the product of  $225n$ , find the largest possible value of  $n$ .

- a) 127                                      b) 133                                      c) 137                                      d) 141

30. Twelve consecutive integers are added together. What is the remainder when the sum is divided by 4?

- a) 1                                      b) 2                                      c) 3                                      d) 4

Tie-Breakers:

1. The grades on 6 tests all range from 0 to 100 inclusive. If the average for the six tests is 93, what is the lowest possible grade on any one test?
2. Find the smallest positive ordered pair of integers  $(x, y)$  if  $11x + 15y = 1$ .
3. Find the largest positive integer  $n$  such that  $2009!$  is divisible by  $10^n$ .