1. **C** For the first hour, the snowmobile travels at 20mph. Hence, at 3PM, there are 265 miles left. Now, since each event has equal probability, the expected value $E(v)$ of the snowmobile's velocity is $20 + \frac{(4k)-(6+k^2)}{2}$. We want to maximize $\frac{dE}{dk}$; hence, $k = 2$, $E(v) = 19$; 265/19 will give $13\frac{18}{19}$ more hours, which is close to 5:00 AM.
2. **C** The "appropriate vertical lines" are $x = 1$ and $x = 10$; hence, for some curve $f(x)$, we need $\int_1^{10} f(x)dx = 1$ (since the area under a probability density function is 1). Only choice C satisfies this.
3. **A** This is just $\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$. 4. **D** $\frac{dv}{dt} = \frac{kv}{t+1} \to \ln(v) = k\ln(t+1) + C \to v = C(t+1)^k; v(0) = 200 \to C = 200; v(3) = 100 \to k = -\frac{1}{2}; 200(8)^{-\frac{1}{2}} = 50\sqrt{2}$.
5. **D** Let x represent the distance of the first snowmobile, y the second, and z the distance between them; after two hours, $x = 200$ and $y = 40$, so $z = \sqrt{40^2 + 200^2} = 40\sqrt{1+25} = 40\sqrt{26}$. Since $x^2 + y^2 = z^2, x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}; 200(50) + 40(20) = 40\sqrt{26}\frac{dz}{dt} \to \frac{5(50)+20}{\sqrt{26}} = \frac{135\sqrt{26}}{13}$.
6. **B** $v(t) = \left\langle \frac{1}{t+1}, 6t, 0 \right\rangle \to$ speed at $t = 1$ is $\sqrt{\frac{1}{4} + 36} = \frac{\sqrt{145}}{2}$
7. **D** From the right, $\frac{x}{[x]}$ goes to ∞ and $\frac{[x]}{x}$ goes to 0, whereas from the left it's vice versa; limit is ∞.
8. **C** $\frac{dN}{dt} = N + 2009 \to \ln(N + 2009) = t + C \to N = Ce^t - 2009; N(0) = 1337 \to C = 3346$. $N(\ln 10) = 33460 - 2009 = 31451$.
9. **B** To find the bounds of the LPE and given Lorenz curve, set $P = Y(P) \to P = 2P^3 - P^3 \to$ $P(P-1)^2 = 0; P = 0, 1$. We want $\dfrac{\int_0^1 PdP - \int_0^1 (2P^2 - P^3)dP}{\int_0^1 PdP} = \dfrac{1}{6}$.
10. **D** I–not necessarily true; the poorest portion of the population could, in theory, own zero wealth. II–not true, if L is constant or linear over an interval then it is neither concave up nor concave down. III–true. IV–true, it makes no sense for the poorest k% of the population to own more than k% of the wealth. V–false. Suppose that, for example, the poorest 50% of the population own 40% of the wealth; then, the poorest 55% cannot own less than 44%, because the extra 5% is richer than the poorest 50. Hence, the function must be concave up (or neither concave up nor concave down, if the poorest portion of the country owns zero wealth).
11. $SA = 4\pi r^2 \to \frac{dA}{dt} = 8\pi r\frac{dr}{dt} \to \frac{dA}{dt} = 8\pi \times 7 \times 2 = 112\pi$.
12. **B** Differentiate to get $-.3x^2 + 4x = 0 \to x(-.3x + 4) = 0 \to x = 0, \frac{40}{3} \to \frac{40}{3}$.
13. The intersection will be at $x = \pi$. Hence $\int_0^\pi (\sin(x) - x + \pi)dx = \dfrac{\pi^2}{2} + 2$.
14. **B** $\frac{dB}{dt} = \frac{1}{t+1} \to B = \ln(t+1); L = t^2 + 2t + 1 \to dL = 2(t+1)dt; 2\int_0^2 (t+1)\ln(t+1)dt =$ $2\int_1^3 u\ln(u)du$. Use integration by parts to obtain $9\ln(3) - 4$, and divide by μ_0.
15. **A** $\int BdL = \xi \to \xi = 100(t^3 + t^2) + C; \dfrac{d\xi}{dt} = 100(3t^2 + 2t)$. Using a step size of 0.1, we have $1314 + (0.1)(100)(3t^2 + 2t) = 1314 + 10(0.23) = 1316.3$.
16. **B** $\dfrac{1}{7-1}\int_1^7 x^2 dx = 19$
17. **E** Choice A fails when one of $\{a_1, a_2, ..., a_k\}$ is -1 because of the $\ln(x)$ obtained; choice B fails because there will be constants in front of the powers of x; C fails when one of $\{a_1, a_2, ..., a_k\}$ is negative; D fails when all of $\{a_1, a_2, ..., a_k\}$ is positive. Hence, none of these statements are true.
18. **A** Any two adjacent sides sum to 10; consider one side to be x and the other to be $10 - x$; $A = x(10 - x) \to$ the average value of these rectangles is $\dfrac{1}{10}\int_0^{10} (10x - x^2)dx = \dfrac{50}{3}$.

19. **D** $A = k^2 \to dA = 4 = 2kdk$. Since the cube maintains its shape, the height must also be increasing; $V = k^3 \to dV = 3k^2dk = \frac{3k}{2}(2kdk) = 6k$.
20. **A** Setting $\sin(x) = 2\cos(x)$ and using $\sin^2(x) + \cos^2(x) = 1$ yields $\sin(x) = -\frac{2}{\sqrt{5}}$ and $\cos(x) = -\frac{1}{\sqrt{5}}$. $r'(x) = 2\cos(2x) + \sec^2(x) = 2(2\cos^2(x) - 1) + 5 = \frac{19}{5}$; multiplying by $\cos(x)$ gives $-\frac{19\sqrt{5}}{25}$.
21. **B** $\int_0^4 k\sqrt{4-x}dx = \dfrac{-2k}{3}(4-x)^{\frac{3}{2}}$ from 0 to 4; this is $\frac{16k}{3} \to k = \frac{3}{16}$. 22. **E** It can be shown that $f(x)$ is its own inverse; $f(x) = g(x) \to f'(x) = g'(x) \to \frac{f'(x)}{g'(x)} = 1$ for all x.
23. **C** The inverse of $f(x)$ can be shown to be $\frac{dx+b}{cx-a}$; $g'(x) = \frac{dcx-da-dcx-bc}{(cx-a)^2} = \frac{-da-bc}{(cx-a)^2}$; to solve the given equation, $da + bc = (cx-a)^2 \to c^2x^2 - 2acx + a^2 - da - bc = 0 \to$ product of roots is $\frac{a^2-ad-bc}{c^2}$.
24. **A** $V = \pi\displaystyle\int_0^1 (\sqrt{x})^2 - (x^2)^2 dx = \pi\int_0^1 x - x^4 dx = \frac{3\pi}{10}$.
25. **D** $11 + \frac{1}{2\sqrt{121}} \times (132 - 121) = 11 + \frac{11}{22} = \frac{23}{2}$
26. **C** Let the first number generated be x and the second be y; then, $xy < e^3, x < e^2, y < e^2$ is the region in which I pay you. The area of this region is $\displaystyle\int_e^{e^2} \frac{e^3}{x}dx + e(e^2) = 2e^3$; hence, the area of the region in which you pay me is $e^4 - 2e^3$. My expected winnings/losses are given by $\dfrac{(e^4 - 2e^3)(2k) - (2e^3)(2k^2+1)}{e^4}$; taking the derivative of the top gives $2(e^4 - 2e^3) - 2e^3(4k)$. Dividing through by $2e^3$ and setting this equal to 0 gives $k = \frac{e-2}{4}$
27. **E** $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{2}{n}\left(\frac{2k^2}{n^2} - 3\right)^2\right)$ would give a definite integral; because of the extra k in the summand, however, this series is divergent.
28. **B** $f''(x) = 12x^2 + 12x - 144 = 12(x-3)(x+4)$; testing regions gives $(-4, 3)$.
29. **C** $x^2 - 2xy + y^2 = 16 \to (x-y)^2 = 16 \to x - y = 4 \to y = x - 4$; $\frac{dy}{dx} = 1$ for all x.
30. **E** $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + ... = 1 + (-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + ...) = 1 - (1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - ...) = 1 - \ln 2$.