The acronym NOTA denotes that "None of These Answers" are correct. The acronym DNE stands for "Does Not Exist". The domain and range of functions are assumed to be either the real numbers or the appropriate subset of the real numbers, unless otherwise stated. $[x]$ represents the greatest integer $\leq x$. $f^{(n)}(x)$ is the n^{th} derivative of $f(x)$ (with respect to x). Good luck, have fun, and may the force be with you!

1. A giant snowmobile full of convicts leaves for Boston, 285 miles away, at exactly 2:00 PM traveling at an initial speed of 20 mph. At each hour, starting at 3:00 PM, either one convict escapes, causing the snowmobile to go $4k$ mph faster for the next hour, or one escaped convict is caught, and the ensuing struggle causes the snowmobile to go $6 + k^2$ mph slower for the next hour. Both situations have equal probability. What is the earliest expected time, to the nearest hour, that this rather interesting snowmobile will reach Boston? (the first thing you want to do is find the value of k that minimizes expected time taken. Also, the snowmobile can go backwards).

A. 11:00 PM B. 12:00 AM C. 5:00 AM D. 6:00 AM E. NOTA

Use the following information for questions 2 and 3:
Grant loves to rant; so much so that he's guaranteed to start ranting every day. However, if he rants 10 times in a day, Michael tapes his mouth shut, causing no more rants for that day. He can also go off on partial rants (i.e. when someone shuts him up during the rant). Let x be the number of rants in a day.

2. Which of the following curves, in conjunction with the x-axis and the appropriate vertical lines, could accurately model the distribution of the probabilities of each possible value of x?

A. $a(x) = \frac{x}{50}$ B. $b(x) = \frac{3x^2}{1000}$ C. $c(x) = \frac{x^2}{333}$ D. $d(x) = x^{\frac{-9}{10}}$ E. NOTA

3. If he can't be stopped in the middle of a rant (i.e. the number of rants is an integer), the probability of an odd number of rants is twice the probability of an even number of rants, the probability of each odd number is the same (i.e. 1, 3, 5, 7, or 9 rants are all equally likely), and the probability of each even number is the same, find the probability he rants 10 times.

A. $\frac{1}{15}$ B. $\frac{2}{15}$ C. $\frac{1}{3}$ D. $\frac{2}{3}$ E. NOTA

4. With little regard for the speed limit, Sida is driving at 200 mph until he sees a police car nearby. He then starts to decelerate, and after 3 minutes his speed is 100 mph. If his rate of deceleration is directly proportional to his current speed (mph) and inversely proportional to (time elapsed since he has started his deceleration plus 1 minute), at what speed will he be traveling after another 4 minutes?

A. 40 mph B. $40\sqrt{5}$ mph C. 50 mph D. $50\sqrt{2}$ mph E. NOTA

5. Two snowmobiles leave the same point in a city two hours apart. The first leaves at 2:00 PM, traveling due south at 50 mph toward New York. The second, after breaking down (hence the delay), creaks along at 20mph due east toward Boston. At what rate is the distance between the two snowmobiles increasing, in mph, at 6:00 PM?

A. $\frac{145\sqrt{26}}{26}$ B. $\frac{165\sqrt{26}}{26}$ C. $\frac{135\sqrt{26}}{13}$ D. $\frac{145\sqrt{26}}{13}$ E. NOTA

6. Somehow, Ari has learned to fly. He flies in the xyz-plane with a position vector of $\langle \ln(t+1), 3t^2, 5 \rangle$, where t is in seconds. Find his speed at $t = 1$.

A. $\frac{\sqrt{97}}{4}$ B. $\frac{5}{4}$ C. $\frac{\sqrt{145}}{2}$ D. $\frac{9}{2}$ E. NOTA

7. $\lim_{x \to 0} \left(\frac{x}{[x]} + \frac{[x]}{x} \right) =$

A. -1 B. 0 C. 2 D. ∞ E. NOTA

8. Because Aneesh is quite impulsive, his favorite number is never the same. The rate of change of his favorite number (where time is measured in seconds) is equal to what his favorite number is at the time plus the numerical value of the year. If his favorite number is currently 1337, what will his favorite number be in $\ln 10$ seconds?

A. 11361 B. 15369 C. 31451 D. 35469 E. NOTA

Use the following information for questions 9-10:
In economics, a country's Lorenz curve displays income inequality. On the x-axis, the cumulative proportion of households is plotted (poorest to richest), and on the y-axis, the cumulative proportion of income is plotted. A perfectly linear Lorenz curve ($Y = P$)is called the line of perfect equality (LPE). The Gini coefficient is the proportion of the total area bound by the LPE and the axes that lies between the LPE and the actual Lorenz curve. For questions 9 and 10, let P be the proportion of people, and let Y be the proportion of wealth they own. Assume a nonnegative Lorenz curve.

9. Suppose that in 2011, the country of Breadworld has a Lorenz curve given by $Y = 2P^2 - P^3$. What would its Gini coefficient be for 2011?

A. $\frac{1}{12}$ B. $\frac{1}{6}$ C. $\frac{5}{12}$ D. $\frac{5}{6}$ E. NOTA

10. Which of the following must be true of any Lorenz curve L?
I. For all P, $Y > 0$
II. L must be concave up
III. $\frac{dL}{dP} \geq 0$
IV. For all P, $Y \leq P$
V. L may be concave up or concave down

A. III, V only B. I, II, III, IV only C. I, III, IV, V only D. III, IV only E. NOTA

11. The radius of a sphere is increasing at 2 cm/sec. What is the rate of change of its surface area (in cm^2/sec) when its radius is 7 cm?

A. 56π B. 112π C. 196π D. 392π E. NOTA

12. A firm's profit when x units of a good are produced is given by $P(x) = -0.1x^3 + 2x^2 + 5$. What value of x maximizes the firm's profit? Round to the nearest integer. A. 11 B. 13
C. 14 D. 16 E. NOTA

13. Compute the area of the region bound above by $y = \sin(x)$, below by $y = x - \pi$, and to the left by $x = 0$.
A. $\frac{\pi^2}{2} - 2$ B. $\frac{\pi}{2} + 2$ C. $\frac{\pi^2}{2}$ D. $\frac{\pi^2}{2} + 2$ E. NOTA

Use the following information for questions 14-15:
In physics, the formula $\int BdL = \mu_0 I$ is often used, where B is the magnetic field (in Tesla), L is distance of a current-carrying wire (in meters), μ_0 is constant, and I is induced current (in amperes).

14. Consider an appropriate setup, with wire currently ($t = 0$) a distance of 1 meter. If the magnetic field is currently 0 but changing at $\frac{1}{t+1}$ T/s and the wire's distance is $t^2 + 2t + 1$ meters, find the total change in induced current from $t = 0$ to $t = 2$.

A. $\frac{9\ln(3)-4}{2\mu_0}$ B. $\frac{9\ln(3)-4}{\mu_0}$ C. $\frac{2}{\mu_0}$ D. $\frac{4}{\mu_0}$ E. NOTA

15. Consider an appropriate setup with wire currently($t = 0$) a distance of 1 meter. Suppose the magnetic field is a constant 100 T, and the wire's distance is changing at $3t^2 + 2t$ m/s. Let $\xi = \mu_0 I$. If $\xi = 1314$ at $t = 0.1$, then use differentials to approximate ξ at $t = 0.2$.

A. 1316.3 B. 1325 C. 1337 D. 1340 E. NOTA

16. Find the average area, in cm^2, of all squares with side lengths between 1 and 7 cm.

A. 16 B. 19 C. 22 D. 25 E. NOTA

17. Define a Raghunandan Function to be be a sum of a finite number of distinct integral powers of x–that is, for integers $a_1 \neq a_2 \neq ... \neq a_k$, such that k is finite, $f(x) = x^{a_1} + x^{a_2} + ... + x^{a_k}$ is a Raghunandan Function. Which of the following is always true?

A. $\int f(x)dx$ is also a Raghunandan Function B. $f'(x)$ is also a Raghunandan Function
C. $\sum_{n=0}^{\infty} f^{(n)}(x)$ converges D. $\sum_{n=0}^{\infty} f^{(n)}(x)$ diverges E. NOTA

18. Find the average area, in m^2, of all rectangles with a perimeter of 20 m.

A. $\frac{50}{3}$ B. 25 C. $\frac{100}{3}$ D. 50 E. NOTA

19. The area of a cross-section of a cube parallel to one of the faces is expanding at 4 cm^2/s. Given that this cube retains its shape, at what rate is its volume increasing, in terms of side length k?

A. $2k$ cm^3/s B. $3k$ cm^3/s C. $4k$ cm^3/s D. $6k$ cm^3/s E. NOTA

20. Let $r(x) = \sin(2x) + \tan(x)$. The sine of a certain third-quadrant angle is two times the cosine of this angle. What is the value of $r'(x)\cos(x)$ when x takes on the value of this angle?

A. $-\frac{19\sqrt{5}}{25}$ B. $-\frac{19}{5}$ C. $\frac{19}{5}$ D. $\frac{19\sqrt{5}}{25}$ E. NOTA

21. Find k such that $y = k\sqrt{4-x}$ (for $0 < x < 4$) is a probability density curve (i.e. find k such that $\int_0^4 k\sqrt{4-x} = 1$.)

A. $\frac{3}{32}$ B. $\frac{3}{16}$ C. $\frac{3}{8}$ D. $\frac{3}{4}$ E. NOTA

22. Let $f(x) = \dfrac{2x+3}{4x-2}$ for $x > \frac{1}{2}$. Let $g(x)$ be the inverse of $f(x)$. Find $\dfrac{f'(2)}{g'(2)}$.

A. -1 B. $-\frac{4}{9}$ C. $\frac{4}{9}$ D. $\frac{1}{2}$ E. NOTA

23. For constant nonzero values of a, b, c, and d, let $f(x) = \dfrac{ax+b}{cx-d}$, where $x \notin \left\{\frac{d}{c}, \frac{a}{c}\right\}$. Let $g(x)$ be the inverse of $f(x)$. Where defined, what is the product of the roots of the equation $\dfrac{1}{g'(x)} + 1 = 0$?

A. $\frac{2a}{c}$ B. $\frac{ad+bc-a^2}{c^2}$ C. $\frac{a^2-ad-bc}{c^2}$ D. $-\frac{ad+bc}{c^2}$ E. NOTA

24. The area bound by the curves $y = x^2$ and $y = x^{\frac{1}{2}}$ in the first quadrant is rotated about the x axis. Find the volume of the solid formed.

A. $\frac{3\pi}{10}$ B. $\frac{\pi}{3}$ C. $\frac{3\pi}{5}$ D. $\frac{2\pi}{3}$ E. NOTA

25. Given that $\sqrt{121} = 11$, use differentials to approximate $\sqrt{132}$.

A. $\frac{264}{23}$ B. $\frac{127}{11}$ C. $\frac{275}{24}$ D. $\frac{23}{2}$ E. NOTA

26. Let's play a game. On my calculator, I generate two random real numbers between 0 and e^2. If the product of the numbers is e^3 or greater, then you pay me \$$2k$; otherwise, I pay you \$$(2k^2+1)$. Because I'm selfish, what value of k will, in the long run, maximize my profits?

A. $\frac{1}{2e-2}$ B. $\frac{1}{e-1}$ C. $\frac{e-2}{4}$ D. $\frac{e-1}{2}$ E. NOTA

27. Evaluate: $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{2k}{n}\left(\frac{2k^2}{n^2}-3\right)^2\right)$.

A. $\frac{32}{5}$ B. $\frac{58}{5}$ C. $\frac{242}{5}$ D. $\frac{422}{5}$ E. NOTA

28. On what interval(s) is $f(x)=x^4+2x^3-72x^2+222x+1$ concave down?

A. (-3, 4) B. (-4, 3) C. $(-\infty,-3)\cup(4,\infty)$ D. $(-\infty,-4)\cup(3,\infty)$ E. NOTA

29. Find $\frac{dy}{dx}$ at (6, 2) given that $x^2+y^2=2xy+16$.

A. -2 B. -1 C. 1 D. 2 E. NOTA

30. Handsome Dan the bulldog has become bulimic after coming too close to a bunch of Hahvahd fans. Yesterday he drank $\frac{1}{2}$ liter of water, but today he threw up $\frac{1}{3}$ of a liter of water. Tomorrow he will drink $\frac{1}{4}$ of a liter of water, but the day after tomorrow he will throw up $\frac{1}{5}$ of a liter of water. If he is immortal, and this continues forever, approximately how many liters of water will this poor bulldog be able to retain in the long run?

A. $\frac{\pi}{4}$ B. $\ln 2$ C. 1 D. ∞ E. NOTA