2009 Limits and Derivatives (Mu)

Solutions:

1. $\lim_{x\to -2} \frac{x^3+8}{x+2} = \lim_{x\to -2} \frac{(x+2)(x^2-2x+4)}{x+2} = \lim_{x\to -2}(x^2-2x+4) = 12$. ***B***

2. $m = -2$, pt(3, -1) $\to y + 1 = -2(x - 3)$; $y = -2x + 5$. ***D***

3. $f(x)$ is undefined at $x = 6$. No limit. ***E***

4. $f(x) = \frac{2x\sin x}{e^x} \to f'(x) = \frac{(2\sin x + 2x\cos x - 2x\sin x)}{e^x}$. ***E***

5. $f(x\cdot) = \sin x - \cos x$ at $(\pi/2, 1)$. $f'(x) = \cos x + \sin x$; $m = 0 + 1 = 1$. $y - 1 = 1\left(x - \frac{\pi}{2}\right) \to y = x - \frac{\pi-2}{2}$. ***A***

6. $f(x) = ax^3 + bx^2 + cx - 1$; $f' = 3ax^2 + 2bx + c$; $f'' = 6ax + 2b \to$ pt. of inflection(0, -1) implies $b = 0$.
   $f' = 3ax^2 + c$; $m = 3$ at $x = 1 \to 3a + c = 3$ and $a + c = -1 \to$ a = 2, c = -3 and $f(x) = 2x^3 - 3x - 1$. ***B***

7. Maximize: $A = (w - 2)(h - 3.5)$; Constraint: $200 = wh \to A = \left(h - \frac{7}{2}\right)\left(\frac{200}{h} - 2\right)$; $A' = \frac{700}{h^2} - 2$; $h = 5\sqrt{14}$. ***C***

8. $f(x) = \frac{x^3-3x^2+3x-5}{x^2-2x+1}$; by division $y = x - 1$. ***D***
9. $\lim_{t\to\infty} v^*(1 - e^{-at}) = v^*(1 - 0) = v^*$. ***C***

10. $f'(x) = \frac{-2x}{\sqrt{1-x^2}}$; $f'' = \frac{-2\sqrt{1-x^2}}{(1-x^2)^2}$. ***C***

11. $f'(x) = \sqrt{1 - x^2} - \frac{x}{\sqrt{1-x^2}}$; $0 = 1 - 2x^2 \to x = \pm\frac{\sqrt{2}}{2} \to f(x) = \pm\frac{\sqrt{3}}{4}$; sum = 0. ***B***
12. $2xdx + 2xy^2dx + 2x^2ydy + 3y^2dy = 0$; $(2x^2y + 3y^2)dy = -(2x + 2xy^2)dx \to \frac{-(2x+2xy^2)}{(2x^2+3y^2)} = \frac{dy}{dx}$. ***A***
13. $\lim_{x\to -\infty} \frac{2-3x-4x^2}{3x^2+6x+10} = \lim_{x\to -\infty} \frac{\frac{2}{x^2} - \frac{3}{x} - 4}{3 + \frac{6}{x} + \frac{10}{x^2}} = -\frac{4}{3}$. ***D***
14. Let $x = h + \frac{\pi}{2}$. $lim_{h\to 0} \frac{tan(2h+\pi)}{h} = lim_{h\to 0} \frac{tan\ 2h + tan\ \pi)}{h(1 - tan\ 2h\ tan\ \pi)} = lim_{h\to 0} \frac{1}{h} \cdot \frac{tan\ 2h + 0}{(1-0)} = lim_{h\to 0} \frac{1}{h} \cdot \frac{sin\ 2h}{cos\ 2h} = lim_{h\to 0} \frac{1}{h} \cdot$
    $2 sinh\ cos\ hcos2h = limh\to 0sinhh \cdot 2\ cos\ hcos2h = 1 \cdot 21 = 2$. ***B***

15. $f'(x) = e^x - 3$; $x = ln3$; $f'(0) < 0$, $f'(2) > 0 \to (ln3, \infty)$. ***C***

16. $f'' = \sin x \to f'(x) = -\cos x + C$; C = -3; $f(x) = -\sin x - 3x + C_1$; $C = 4 \to f(x) = -\sin x - 3x + 4$. ***A***

17. $f'(t) = 3at^2 + 2bt + c$; $f''(t) = 6at + 2b$; $0 = 6at + 2b \to t = -\frac{b}{3a}$. ***B***

18. $t(d) = \frac{\sqrt{x^2+4}}{3} + \frac{5-x}{4}$; $t'(x) = \frac{x}{3\sqrt{x^2+4}} + \frac{5-x}{4}$; $x = \frac{6}{\sqrt{7}} \to 5 - x = \frac{35-6\sqrt{7}}{7}$. ***D***
19. $\lim_{x\to 0^+} \frac{\sqrt{1+\frac{1}{x^2}}}{\frac{1}{x}} = \lim_{x\to 0^+} \sqrt{1 + x^2} = 1$. ***C***

20. ***E***

21. $f'(x) = \frac{1}{3}(x^2 + 3x - 1)^{-2/3}(2x + 3)$; $f'(-1) < 0$, $f'(0) > 0$, min at $x = -\frac{3}{2}$, $f(-3/2) = -\frac{\sqrt[3]{26}}{2}$. ***C***

22. $4 = w^2h \rightarrow h = \frac{4}{w^2}$; $SA = w^2 + \frac{16}{w}$; $SA' = 2w - \frac{16}{w^2}$; $0 = 2w^3 - 16 \rightarrow SA = 12$. ***C***

23. Eliminate b and d because $f(0) \neq 0$, eliminate a because domain of graph is (-1, 1). ***C***

24. $g' = -\frac{3}{x^2} + \frac{2}{x^3} = m \rightarrow m_2 = -\frac{1}{2}$. $y - \frac{9}{4} = -\frac{1}{2}(x - 2) \rightarrow 2x + 4y = 13$. ***A***

25. $R' = \frac{225}{\sqrt{x}}$; $x = 16 \rightarrow R' = 56.25$ ***C***

26. Omit

27. $f'(x) = 25x^4 - 6x + 1$; $f'(0) > 0$, $f'(.3) < 0$, $f'(1) > 0$. $f''(x) = 100x^3 - 6$, 1 pt of inflection so 2 local max/min. ***C***

28. $4x^3 + y^3 + 3y^2x\frac{dy}{dx} = 0$; $\frac{dy}{dx} = \frac{-4x^3 - y^3}{3y^2x}$; $\frac{d(-1,1)}{dx} = -1$. ***B***

29. $y' = -\frac{1}{x^2}$; $y = -\frac{1}{x^2}(x - 4)$; $y = -\frac{1}{x} + \frac{4}{x^2}$; $-\frac{1}{x} + \frac{4}{x^2} = \frac{1}{x}$; $x = 2$; Pts:(2, ½) and (4, 0); $y = -1/4x + 1 \rightarrow$
$A\Delta = ½(1)4 = 2$. ***B***

30. $\lim_{x\to 0^+} \frac{2\sin x\cos x - 2\sqrt{x}\sin x + x^2}{x} = 2\cos(0) - 2\sqrt{0} + 0 = 2$. ***D***

Tie-Breakers:

1. $y = \frac{2}{x} - 1$; $y' = -\frac{2}{x^2}$; $y'' = \frac{4}{x^3}$

2. $f'(x) = -\sqrt{1 + 3x^2} - \frac{x}{\sqrt{1+3x^2}}$. $m = -\frac{9}{4}$; $y = -\frac{9}{4}x + \frac{1}{4}$

3. $\lim_{x\to\infty} \frac{\sqrt{2+4x^2}}{x} = 2$; $\lim_{x\to-\infty} \frac{\sqrt{2+4x^2}}{x} = -2$