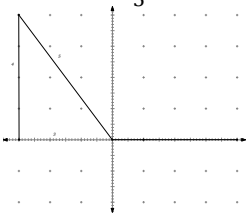


TRIGONOMETRY TOPIC TEST 2009 (ALPHA)

Solutions

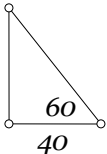
1.  $\sec \theta = -\frac{5}{3}$ . **C**



2.  $8^2 - 3^2 = 55$ .  $y = \sqrt{55}$ ;  $\tan \theta = -\frac{\sqrt{55}}{3}$ . **B**

3.  $\frac{1+\sec\theta}{\tan\theta + \sin\theta} = \frac{\cos\theta + 1}{\sin\theta + \sin\theta\cos\theta} \rightarrow \frac{\cos\theta + 1}{\sin\theta + \sin\theta\cos\theta} = \frac{1}{\sin\theta} \rightarrow \csc \theta$ . **B**

4.  $(40)\sqrt{3}$  m. **A**



5.  $2(\sin(x) + \cos(x)) < \sqrt{1 + \sin 2x} \rightarrow \sin 2x + 1 \geq 0$ , which is only true when  $\sin x + \cos x < 0$ .  
But  $\sin x + \cos x = \sqrt{2} \sin(x + 45^\circ)$ , so  $135^\circ < x < 315^\circ$ . **B**

6.  $\sin\left(\tan^{-1}\left(\frac{15}{8}\right) + \cos^{-1}\left(\frac{4}{5}\right)\right) = \sin(a + b) = \sin a \cos b + \cos a \sin b \rightarrow \left(\frac{15}{17}\right)\left(\frac{4}{5}\right) + \left(\frac{8}{17}\right)\left(\frac{3}{5}\right) = \frac{84}{85}$ . **C**

7.  $3\tan^2\mu - 1 = 0$ ;  $\tan \mu = \pm\frac{\sqrt{3}}{3} \rightarrow \mu = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \rightarrow 4\pi$ . **A**

8.  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$ ; If  $\csc A = \frac{3}{2}$  then  $\tan 2A = \frac{2\left(\frac{2}{\sqrt{5}}\right)}{1 - \frac{4}{5}} = -4\sqrt{5}$ . **C**

9.  $\frac{\cos \varphi}{1 - \sin \varphi} - \frac{1 - \sin \varphi}{\cos \varphi} = \frac{\cos^2 \varphi - (1 - \sin \varphi)^2}{\cos \varphi (1 - \sin \varphi)} = \frac{\cos^2 \varphi - 1 + 2 \sin \varphi - \sin^2 \varphi}{\cos \varphi (1 - \sin \varphi)} = \frac{1 - \sin^2 \varphi - 1 + 2 \sin \varphi - \sin^2 \varphi}{\cos \varphi (1 - \sin \varphi)}$ ;  
 $\frac{2 \sin \varphi (1 - \sin \varphi)}{\cos \varphi (1 - \sin \varphi)} = 2 \tan \varphi$ . **C**

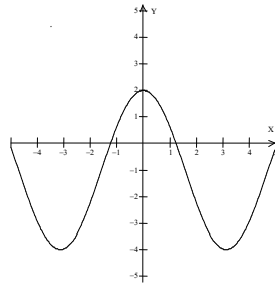
10.  $\left(\cos \frac{11\pi}{6} + \sin \frac{\pi}{6}\right)\left(\tan \frac{\pi}{6} + \cot \frac{4\pi}{3}\right) = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)\left(\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\right) = \frac{3+3\sqrt{3}}{3}$ . **D**

11.  $2\cos 3x - 2\sin 3x$ , where  $a = 2$ ,  $b = 2$ ,  $B = 3$  and  $A = \sqrt{8} = 2\sqrt{2}$ ,  $\tan C = 1$  and  $C = \frac{\pi}{4}$ .

$2\cos 3x - 2\sin 3x = 2\sqrt{2}\cos\left(3x - \frac{\pi}{4}\right)$ . Amp.  $= 2\sqrt{2}$ , Period  $= \frac{2\pi}{3}$ , P.S.  $= \frac{\pi}{12}$ . **C**

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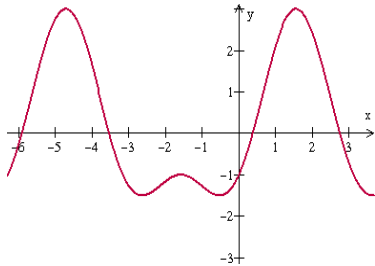
12.  $f(x) = 3\cos x - 1$ . **B**



13. Using right triangles:  $3\cos A = \sqrt{5}$ ;  $5\sin B = \sqrt{15}$ . Then  $3\cos A + 5\sin B = \sqrt{5} + \sqrt{15}$ . **C**

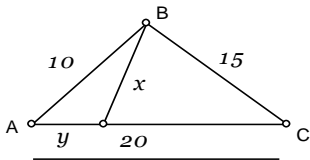
14.  $\sin 3t - \sin 7t = 2 \cdot \frac{\cos[(3t+7t)]}{2} \cdot \frac{\sin[(3t-7t)]}{2} \rightarrow 2\cos(5t)\sin(-2t) = -2\cos(5t)\sin(2t)$ . **D**

15. Maximum = 3 and minimum = -1.5 and Amp =  $\frac{3 - (-1.5)}{2} = 2.25$ ; Period =  $.3 - (-5.9) = 6.2$ . **B**



16.  $\frac{\sin Q}{q} = \frac{\sin R}{r}$ ;  $\frac{1}{6} = \frac{\frac{\sqrt{5}}{3}}{r} \rightarrow \frac{1}{2}, r = 2\sqrt{5}; r = 4\sqrt{5}$ . **D**

17. Area =  $\frac{1}{2}bc \sin A$ ;  $\cos A = \frac{400+100-225}{400} = \frac{11}{16}$ ;  $\sin A = \frac{3\sqrt{15}}{16}$ ; Area =  $\frac{20 \cdot 10}{2} \cdot \frac{3\sqrt{15}}{16} = \frac{75\sqrt{15}}{4}$ ; **C**



18.  $x^2 = 25 + 100 - 100\left(\frac{11}{16}\right) \rightarrow x^2 = \frac{225}{4}; x = \frac{15}{2}$ . **D**

19.  $\frac{2^{4\sin x}}{2^{\cos^2 x}} = 2^{\sin^2 x + 1} \rightarrow 2^{4\sin x - \cos^2 x} = 2^{\sin^2 x + 1}; 4\sin x - \cos^2 x = \sin^2 x + 1$   
 $\sin^2 x + \cos^2 x - 4\sin x + 1 = 0; 2 - 4\sin x = 0 \rightarrow \sin x = 1/2, x = \frac{\pi}{6}, \frac{5\pi}{6} \dots$  **B**

20.  $\cos Q = \frac{\sqrt{7}}{4} \rightarrow \sin Q = \frac{3}{4}; \cos R = -\frac{2}{3} \rightarrow \sin R = \frac{\sqrt{5}}{3}; \frac{3}{4} = \frac{\frac{\sqrt{5}}{3}}{r} \rightarrow r = \frac{8\sqrt{5}}{3}$ . **A**

21.  $\cos[\tan^{-1}(-\frac{\sqrt{3}}{3})]. \tan^{-1}(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6}; \cos(-\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ . **C**

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22.  $\sin(6x) + \cos(4x) = \sin(5x + x) + \cos(5x - x) \rightarrow$   
 $\sin(5x)\cos(x) + \sin(x)\cos(5x) + \cos(5x)\cos(x) + \sin(5x)\sin(x) = 0 \rightarrow \sin(5x)(\cos(x) + \sin(x)) +$   
 $\cos(5x)(\sin(x) + \cos(x)) \rightarrow (\sin(5x) + \cos(5x))(\sin(x) + \cos(x)). \rightarrow \tan(x) = -1$  and  $x = 135^\circ$ ;  
 $\tan(5x) = -1$  so  $5x = 135 + 180k$  and  $x = 27^\circ + 36k \rightarrow 27^\circ, 53^\circ, 99^\circ, 135^\circ,$  and  $171^\circ$  **C**
23.  $(1 - i)^5; r = \sqrt{2}; \theta = \tan^{-1}(-1) = -\frac{\pi}{4}; (\sqrt{2}^5) \left( \cos\left(-\frac{5\pi}{4}\right) + i\sin\left(-\frac{5\pi}{4}\right) \right) =$   
 $4\sqrt{2} \left( \cos\left(\frac{5\pi}{4}\right) - i\sin\left(\frac{5\pi}{4}\right) \right)$ . **B**
24.  $x = 4\cos(30^\circ) = 2\sqrt{3}; y = 4\sin(30^\circ) = 2$  so  $A(4; 30^\circ) = A(2\sqrt{3}, 2); x = -6\cos(60^\circ) = -3;$   
 $y = -6\sin(60^\circ) = -3\sqrt{3}$ , so  $B(-6; 60^\circ) = B(-3, -3\sqrt{3})$ .  $d = \sqrt{(2\sqrt{3} + 3)^2 + (2 + 3\sqrt{3})^2} \rightarrow$   
 $2\sqrt{13 + 6\sqrt{3}}$ . **D**
25.  $\sin^4x + 2\sin^2\cos^2 + \cos^4x - 2\sin^2\cos^2 = (\sin^2x + \cos^2x)^2 - 2\sin^2\cos^2 = 1 - 2\sin^2x\cos^2x;$   
 $2\sin x \cos x = \frac{24}{25} \rightarrow 4\sin^2x\cos^2x = \frac{576}{625} \rightarrow \sin^2x\cos^2x = \frac{144}{625} \rightarrow 1 - \frac{288}{625} = \frac{337}{625}$ . **D**
26.  $2 \sec 2x = \tan y + \cot y \rightarrow \frac{2}{\cos 2x} = \frac{\sin y}{\cos y} + \frac{\cos y}{\sin y}; \frac{2}{\cos 2x} = \frac{1}{\cos y \sin y}; 2\sin y \cos y = \sin 2y = \cos 2x$ .  
 Least value for  $2x + 2y = 90$  and  $x + y = 45$ . **B**
27.  $(\cos x + \sin x)^2 = \frac{25}{16}; 2\sin x \cos x = \frac{9}{16}; \sin 2x = \frac{9}{16} \rightarrow \cos 2x = \frac{5\sqrt{7}}{16} = (\cos x + \sin x)(\cos x - \sin x),$   
 $\frac{5\sqrt{7}}{16} = \frac{5}{4}(\cos x - \sin x) \rightarrow \cos x - \sin x = \frac{\sqrt{7}}{4}$ . **C**
28.  $y = 2\cos(3x - \frac{\pi}{2})$ , period =  $\frac{2\pi}{3}$ , p.s. =  $\frac{\pi}{6} \rightarrow 4$ . **B**
29.  $\sin 2x(\tan 2x + 1) = 0; \sin 2x = 0, 2x = \sin^{-1} 0 \rightarrow x = 0, \frac{\pi}{2}; 2x = \tan^{-1} -1 \rightarrow x = \frac{5\pi}{8}, \frac{7\pi}{8}$ . **E**
30.  $30^\circ$  for a) 0, F, b)  $\frac{\sqrt{3}+1}{2}$ , F, c) 1, T, d)  $\frac{\sqrt{3}+1}{2}$  F,  $90^\circ$  for a) -2 F, b) 0, T, c) 0, T, d) -1, F. **C**

Tie-Breakers:

1.  $1 - 2\sin^2x = \frac{\sqrt{3}}{4}. \sin^2x = \frac{4 - \sqrt{3}}{8}; \sin x = \pm \sqrt{\frac{4 - \sqrt{3}}{8}} = \frac{\sqrt{8 - 2\sqrt{3}}}{4}$

2.  $\sin 120^\circ = \frac{\sqrt{3}}{2}; A = 30\left(\frac{\sqrt{3}}{2}\right) = 15\sqrt{3}$

3.  $\frac{\sec A}{\sin A} - \frac{\sin A}{\cos A} \rightarrow \frac{1 - \sin^2 A}{\sin A \cos A} = \frac{\cos^2 A}{\sin A \cos A} \rightarrow \cot A$ .