	Mu Ciphering Answers
0.	$\frac{1}{11}$
1.	$\frac{1}{4}$
2.	$\frac{\sqrt{3}}{2}e^{\frac{1}{4}}$
3.	$\frac{\sqrt{5}}{5}$
4.	30
5.	0
6.	32
7.	$\frac{1}{3}$
8.	$3\sqrt{2}$
9.	$\frac{1296}{35}$
10.	$\frac{1}{e}$

The following were changed at the resolution center at the convention: # 4 90, # 9 36.

0. Let $u = 1\text{-}x^2$; then $\int_0^1 2x(1-x^2)^{10}\,dx = -\int_1^0 u^{10}\,du = \int_0^1 u^{10}\,du = 1/11$.

1. $f(x) = \sum_{n=1}^{\infty} x^n = \frac{x}{1-x}$; $f'(x) = \frac{1}{(1-x)^2} \Rightarrow \frac{f(x)}{f'(x)} = x(1-x) = x - x^2$. This is at a maximum when $x = ½$ → plugging in ½ gives ½ – ¼ = ¼.

2. Rewrite $f(x)$ as $e^{0.5\sin(2x)\tan(x)} = e^{\sin(x)\cos(x)\tan(x)} = e^{\sin^2(x)}$. Then $f'(x) = 2\sin(x)\cos(x)e^{\sin^2(x)} = \sin(2x)e^{\sin^2(x)}$. Plugging in, $f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}e^{1/4}$.

3. Let z denote the distance between them. Then $z = \sqrt{x^2+y^2} \Rightarrow \frac{dz}{dt} = \left(\frac{1}{\sqrt{x^2+y^2}}\right)\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$. After one minute (aka sixty seconds), we have $x = 12$, $y = 24$. Thus, $\frac{dz}{dt} = \left(\frac{1}{\sqrt{12^2+24^2}}\right)\left(12(0.2)+24(0.4)\right) = \frac{12}{12\sqrt{5}} = \frac{\sqrt{5}}{5}$.

4. This can be done with determinants and ugly expressions, but note that *the height of the triangle is always 5* and so we can essentially disregard the value of $3a^3 - a^2 + 2a - 4$. Hence, we're only concerned with the length of the base—which is the distance from (0, 0) to ($a^2 + 1$, 0), or $a^2 + 1$. Hence the triangle's area is $A = \frac{1}{2}(5)(a^2+1) \Rightarrow \frac{dA}{da} = 5a = 5(6) = 30$.

5. Let $u = x^2$. Then $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} x^3 e^{x^2}\,dx = \int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} x\left(x^2 e^{x^2}\right)dx = 1/2 \int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} 2x\left(x^2 e^{x^2}\right)dx = \int_{\ln 2}^{\ln 3} ue^u\,du$.
Use integration by parts on this to get $\left[ue^u - e^u\right]_{\ln 2}^{\ln 3} = 3\ln 3 - 2\ln 2 - 1; A + B + C = 0$.

6.2001 = 3*667 = 3*23*29; A + B = 32.

7. $6x + y^2 + 2xy\frac{dy}{dx} - 3y^2\cos(x)\frac{dy}{dx} + y^3\sin(x) = 0$; plugging in, $4 - 12\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1}{3}$.

8. Apply L'Hopital's Rule first: $\lim_{x\to 0}\frac{\int_0^{\sin(3x)} e^{2t}\,dt}{\sqrt{1-\cos(x)}} = \lim_{x\to 0}\frac{3\cos(3x)e^{2\sin(3x)}}{\frac{\sin x}{2\sqrt{1-\cos(x)}}}$.

Now for some algebra:

$$\lim_{x\to 0}\frac{3\cos(3x)e^{2\sin(3x)}}{\frac{\sin x}{2\sqrt{1-\cos(x)}}} = \lim_{x\to 0}\frac{6\cos(3x)e^{2\sin(3x)}}{\frac{\sqrt{1-\cos^2(x)}}{\sqrt{1-\cos(x)}}} = \lim_{x\to 0}\frac{6\cos(3x)e^{2\sin(3x)}}{\sqrt{\frac{(1-\cos(x))(1+\cos(x))}{1-\cos(x)}}} = \lim_{x\to 0}\frac{6\cos(3x)e^{2\sin(3x)}}{\sqrt{1+\cos(x)}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}.$$

9. If expected winnings are zero, then *k* must be expected cost. The probability of winning (denoted $P(n)$) on day *n* is the probability of getting double sixes on day *n* but not on any day before; since the probability of a double six is $\frac{1}{36}$, this is $\left(\frac{35}{36}\right)^{n-1}\left(\frac{1}{36}\right) = \left(\frac{1}{35}\right)\left(\frac{35}{36}\right)^{n}$. Since it costs \$1 each day to play, the cost of winning on day *n* is just \$n. Hence, our expected cost is

$$\sum_{n=1}^{\infty} nP(n) = \frac{1}{35}\sum_{n=1}^{\infty} n\left(\frac{35}{36}\right)^n = \frac{1}{35}\left(\frac{1}{\left(1-\frac{35}{36}\right)^2}\right) = \frac{1}{35}\left(36^2\right) = \frac{1296}{35}.$$

10. The probability that Ms. Herron doesn't get her hat back on a given day is $\frac{n-1}{n}$. Hence,

$$P(n) = \left(\frac{n-1}{n}\right)^n = \left(1-\frac{1}{n}\right)^n \Rightarrow \lim_{n\to\infty} P(n) = \frac{1}{e}.$$