

Answer Key:

Section I – Trivia 101

1. 79 (THE WHO)
2. 86 (HIGHLANDER)
3. 77 (THE OFFICE)
4. 317 (THE LION, THE WITCH AND THE WARDROBE)
5. 160 (ROLLING STONE)
6. 100 (THE ONION)
7. 137 (WES ANDERSON)
8. 252 (THE CRITERION COLLECTION)
9. 139 (CHUCK PALAHNIUK)
10. 349 (YOSHIMI BATTLES THE PINK ROBOTS)

Section II – What Number am I?

1. 22
2. 132
3. 98
4. 77
5. 8
6. 159
7. 58
8. 34
9. 105
10. 89

Section III – Unscramble Me

1. OCTAHEDRON (TETRAHEDRON, HEXAHEDRON, DODECAHEDRON, ICOSAHEDRON)
2. RATIONAL or WHOLE (NATURAL, INTEGER, REAL, COMPLEX)
3. TWO (ONE, THREE, FOUR)
4. CALCULATOR (ABACUS, SLIDE RULE)
5. GEOMETRIC (ARITHMETIC, HARMONIC)
6. MODE (MEAN, MEDIAN)
7. MU (ALPHA, THETA)
8. ZERO or NEUTRAL (POSITIVE, NEGATIVE)
9. RING (GROUP, INTEGRAL DOMAIN, FIELD)
10. CALCULUS (ALGEBRA, GEOMETRY, STATISTICS)

Section IV – Somewhat Difficult Problems

1. 9 (RSS, SRS, SSR, PRR,RPR, RRP, SPP, PSP, PPS)

2. 1979 (Any number from a perfect square n^2 to $n(n+1)-1$, inclusive, will suffice for c . Since $44^2 = 1936$ and $45^2 = 2025$, all numbers from 1936 to $44 * 45 - 1 = 1979$ will suffice for c , and the next number that has a solution will be 2025. The answer, FYI, is $x = 1979 / 44$)

3. 5 ($\langle 1,16 \rangle$; $\langle 1,4,16 \rangle$; $\langle 1,-4,16 \rangle$; $\langle 1,2,4,8,16 \rangle$; $\langle 1,-2,4,-8,16 \rangle$)

4. $-4/3$ (Since $A+B = -B/A$, $A^2 + AB + B = 0$, and solving for B gives $B = -A^2 / (A+1)$. Since $AB = -16 / (27A)$, $A^4 / (A+1) = 16 / 27$. The only rational solution to this is $A = -2/3$. Plugging back in gives $B = -4/3$)

5. 1 (The first term must be 2, and then the remaining terms are 3, 5, 7, 9, ..., $2n-1$,.... Therefore, the only even term is the first one.)

6. 11 (This sequence looks like this: 2, 6, 4, 7, 11, 8, 14, 8, 8, 11, 14, 11, and then it repeats beginning with the first 11. So there are 4 terms initially, with a repeating cycle of 8 terms after that. Since $2012 = 4 + 251 * 8$, the 2012th term is the last 11 in the cycle, meaning the 2010th term is the 11 two terms before it.)

7. 15 (Since the sum of the first n positive integers is $n(n+1)/2$, we are looking for the smallest value of n such that $n(n+1) = 2^a * 3 * k$ for some positive integer k . Since $16 = 2^4$, and 15 is divisible by 3, the smallest value of n is 15.)

8. 20 (By squaring both sides, $2a + 2\sqrt{a^2 - 80} = b + 4\sqrt{5}$. Since a and b are positive integers, $2\sqrt{a^2 - 80} = 4\sqrt{5}$, or $a = 10$. Since $2a = b$, $b = 20$.)

9. 3995 ($X = 1 + 2011 + (2012)(2011) + (2013)(2012)(2011) \dots$, so $x_{2011} = 1$, $x_{2012} = 0$, $x_{2013} = 1$, $x_{2014} = 2012$, $x_{2015} = 9$, and $x_{2016} = 1972$. $1+0+1+2012+9+1972=3995$)

10. 29/4165 (There are $52 * 51 * 50 * 49 = 6497400$ different sequences of four cards, but when dealing to two people, the first person's cards "switched" in the order doesn't matter. Thus there are only 1/4 of these that are possible, so the denominator is $6497400/4 = 1624350$. For the numerator, if both cards are the same, there are 13 different ranks, along with 6 different combinations of two suits for the first person (the second person's suits are determined automatically once the first person's suits are determined), so there are 78 ways for both cards to have the same rank. If the two cards in each hand

have different ranks, there are $\binom{13}{2} = 78$ different choices of two cards, and then there are $4 * 3 = 12$

different combinations of one card, and $4 * 3 = 12$ combinations of the other, so there are a total of $78 * 12 * 12 = 11232$ total ways to distribute the cards in this way. Thus, the total numerator is $78 + 11232 = 11310$, and the fraction is $11310/1624350$, which reduces to 29/4165.)

11. 84966 (This is the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ..., where the positive integer n is repeated n times. Thus, the series is 1 1, 2 2s, 3 3s, 4 4s, ..., 62 62s, and 57 63s. The sum is thus

$1^2 + 2^2 + 3^2 + \dots + 62^2 + 57 * 63 = 62 * 63 * 125 / 6 + 3591 = 84966$.)

12. 1, 5, and 9 (The Pythagorean triples are in the form $a^2 - b^2, 2ab, a^2 + b^2$. The first such triple is 3,4,5 ($a=2, b=1$). Succeeding such triples come from a recursive definition where $a_{n+1} = 2a_n + b_n$ and $b_{n+1} = a_n$. Thus, the next triples are 21,20,29 ($a=5, b=2$); 119,120,169 ($a=12, b=5$); 697,696,985 ($a=29, b=12$); and 4059,4060,5741 ($a=70, b=29$). The last digits of the hypotenuse lengths form the sequence 5,9,9,5,1,1,5,9,9,5,1,1,5,9,9,5,1,...)

13. 9 (Let n be the common integer root. Then $n^3 - An - B = 0 = n^2 - An + B$. Therefore, $B = (n^3 - n^2) / 2$, and the smallest odd value of B is 9, which occurs when $n = 3$.)

14. 54 (Powerful numbers can be written in the form a^2b^3 , where a and b are positive integers. Of the first 1000 positive integers, the ones that are powerful are 1, 4, 8, 9, 16, 25, 27, 32, 36, 49, 64, 72, 81, 100, 108, 121, 125, 128, 144, 169, 196, 200, 216, 225, 243, 256, 288, 289, 324, 343, 361, 392, 400, 432, 441, 484, 500, 512, 529, 576, 625, 648, 675, 676, 729, 784, 800, 841, 864, 900, 961, 968, 972, 1000, and there are 54 of these.)

15. 158 ($n = 2^{n_1} 3^{n_2} 11^{n_3} 13^{n_4}$, where n_1 is an odd multiple of 3, 11, and 13 (hence, $n_1 = 429$); n_2 is an even multiple of 11 and 13, and one more than a multiple of 3 (hence, $n_2 = 286$); n_3 is an even multiple of 3 and 13, and one more than a multiple of 11 (hence, $n_3 = 78$); and n_4 is an even multiple of 3 and 11, and one more than a multiple of 13 (hence, $n_4 = 66$). The sum is thus

$$-(2 - 429) + (3 - 286) - (11 - 78) + (13 - 66) = 158.$$

Section V – Minority Game

Answers to be determined after test is given.

Section VI – Game Theory

1. 98 coins for head pirate, 1 coin for third pirate, 1 coin for fifth pirate
2. Both prisoners should defect from the other
3. 0
4. The bidding will increase indefinitely
5. \$99 for first person, \$1 for second person

Section VII – Sum of Four Perfect Squares

For example, there are others:

101 = 1+100	114 = 1+49+64	127 = 1+1+25+100	139 = 9+9+121
102 = 1+1+100	115 = 1+1+49+64	128 = 64+64	140 = 4+36+100
103 = 1+1+1+100	116 = 16+100	129 = 4+25+100	141 = 1+4+36+100
104 = 4+100	117 = 36+81	130 = 9+121	142 = 1+4+16+121
105 = 1+4+100	118 = 1+36+81	131 = 1+9+121	143 = 9+9+25+100
106 = 25+81	119 = 1+1+36+81	132 = 16+16+100	144 = 144
107 = 1+25+81	120 = 4+16+100	133 = 1+16+16+100	145 = 1+144
108 = 1+1+25+81	121 = 121	134 = 9+25+100	146 = 9+16+121
109 = 9+100	122 = 1+121	135 = 1+9+25+100	147 = 1+9+16+121
110 = 1+9+100	123 = 1+1+121	136 = 36+100	148 = 4+144
111 = 1+4+25+81	124 = 1+1+1+121	137 = 1+36+100	149 = 49+100
112 = 16+16+16+64	125 = 25+100	138 = 1+1+36+100	150 = 25+25+100
113 = 49+64	126 = 1+25+100		

Section VIII – The 2010 Game

For example, there are others (probably):

1 = 1 ² + 0 + 0	18 = 20 - 1 - 0!	35 = 12!!!!!!!- 0!+ 0
2 = 2 + 0 ¹ + 0	19 = 21 - 0! - 0!	36 = 12!!!!!!!+ 0 + 0
3 = 2 + 1 + 0 + 0	20 = 20 + 0 ¹	37 = 12!!!!!!!+ 0! + 0
4 = 2 + 1 + 0! + 0	21 = 21 + 0 + 0	38 = 12!!!!!!!+ 0! + 0!
5 = 2 + 1 + 0! + 0!	22 = 21 + 0! + 0	39 = (12 + 0! + 0)!!!!!!!
6 = (2 + 1)! + 0 + 0	23 = 21 + 0! + 0!	40 = 10!!!!!!!+ 0 ²
7 = (2 + 1)! + 0! + 0	24 = ((2 + 1)! + 0! + 0)!!!!	41 = 10!!!!!!!+ 2 ⁰
8 = (2 + 1)! + 0! + 0!	25 = (12 + 0)!!!!!!!- 0!	42 = (12 + 0! + 0)!!!!!!!
9 = 10 - 2 ⁰	26 = (12 + 0! + 0)!!!!!!!	43 = (12 - 0)!!!!!!!- 0!
10 = 20 - 10	27 = (10 - 2 ⁰)!!!!!!!	44 = (12 - 0! + 0)!!!!!!!
11 = 10 + 2 ⁰	28 = (12 + 0! + 0)!!!!!!!	45 = (10 - 2 ⁰)!!!!
12 = 12 + 0 + 0	29 = 10!!!!!!!- 2 + 0!	46 = (21 + 0! + 0)!!!!!!!
13 = 12 + 0! + 0	30 = 20 + 10	47 = 12!!!!!!!- 0! + 0
14 = 12 + 0! + 0!	31 = 10!!!!!!!+ 2 - 0!	48 = 12!!!!!!!+ 0 + 0
15 = (2 + 1 + 0! + 0!)!!	32 = ((2 + 1)! + 0! + 0)!!!!	49 = 12!!!!!!!+ 0! + 0
16 = ((2 + 1)! + 0! + 0)!!!!!!	33 = (10 + 2 ⁰)!!!!!!!	50 = 12!!!!!!!+ 0! + 0!
17 = ((2 + 1)!)!! - 0! + 0	34 = (((2 + 1)!)!! - 0! + 0)!!!!!!!	

Section IX – Cryptograms

1. There is nothing strange in the circle being the origin of any and every marvel. -Aristotle
2. I don't believe in mathematics. -Albert Einstein
3. Mathematics is a game played according to certain rules with meaningless marks on paper.
-David Hilbert
4. A mathematician is a blind man in a dark room looking for a black cat which isn't there.
-Charles Darwin
5. Mathematics is the only science where one never knows what one is talking about nor whether what is said is true. -Bertrand Russell

Section X – Wild Card Question

Answer to be determined after test is given.