

For all questions, answer E. "NOTA" means none of the above answers is correct.

- 1 The sum of the squares of two positive numbers is 200. What is the maximum product of the two numbers?

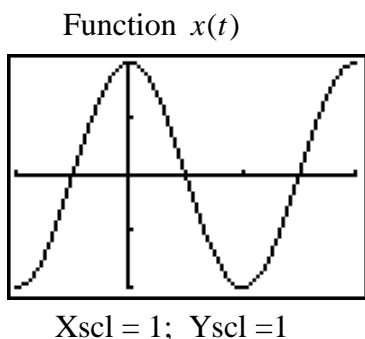
A) 100 B) $25\sqrt{7}$ C) 28 D) $24\sqrt{14}$ E) NOTA

2. The circumference of a circle is increasing at a constant rate of 3π inches per second. At the moment the circle has an area of 9π square inches, the rate of change in the area of the circle is

A) $3/2 \text{ in}^2/\text{sec}$ B) $9\pi \text{ in}^2/\text{sec}$ C) $9\pi^2 \text{ in}^2/\text{sec}$ D) $\frac{9\pi}{4} \text{ in}^2/\text{sec}$

E) NOTA

3. Given the graph of the position function $x(t)$ of a particle moving back and forth on the x-axis and having points $(0, 2)$ and $(4, 2)$, the equation representing velocity of the particle as a function of time is



- A) $v(t) = -4 \sin 2t$
 B) $v(t) = 2 \cos 2t$
 C) $v(t) = -2 \sin \pi t$
 D) $v(t) = -2\pi \sin \pi t$
 E) NOTA

4. Water is being poured into a conical paper cup at the rate of 15 inches^3 per minute. The height of the paper cup is 20 inches and its diameter is 10. At what rate is the surface of the water rising at the time when its height is 8 inches?

A) $\frac{15}{2} \text{ in}/\text{min}$ B) $\frac{15}{2\pi} \text{ in}/\text{min}$ C) $\frac{5}{4\pi} \text{ in}/\text{min}$ D) $\frac{15}{4\pi} \text{ in}/\text{min}$ E) NOTA

5. Jacob is a bacteria farmer and the rate at which his bacteria grow is directly proportional to the amount of bacteria present. At time $t = 0$, there are 20 bacteria present. If there are 1,620 bacteria present after 4 days, how many bacteria are present after 6 days?

A) 7,290 B) 12,640 C) 14,580 D) 21,870 E) NOTA

6. A particle moves along the x-axis so that its velocity is $v(t) = \sin t + \frac{1}{2}$ for $0 \leq t < 2\pi$. Find the interval(s) in which the speed of the particle is decreasing

A) $\frac{7\pi}{6} < t < \frac{11\pi}{6}$

B) $\frac{\pi}{2} < t < \frac{7\pi}{6}, \frac{3\pi}{2} < t < \frac{11\pi}{6}$

C) $\frac{\pi}{2} < t < \frac{3\pi}{2}$

D) $\frac{7\pi}{6} < t < \frac{3\pi}{2}$

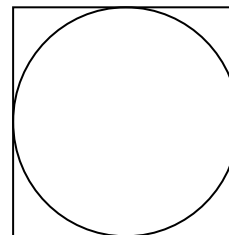
E) NOTA

For Questions #7 and 8:

A circle is inscribed in a square as shown in the figure. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency.

7. Find the rate at which the perimeter of the square is increasing.

A) $\frac{2}{\pi}$ in/sec B) $\frac{12}{\pi}$ in/sec C) $\frac{24}{\pi}$ in/sec D) $\frac{48}{\pi}$ in/sec E) NOTA



8. At the instant when the area of the circle is 25π sq. inches, find the rate of increase in the area enclosed between the circle and the square.

A) $\frac{60}{\pi} - 24$ in²/sec B) $\frac{60}{\pi} - 30$ in²/sec C) $\frac{120}{\pi} - 24$ in²/sec

D) $\frac{120}{\pi} - 30$ in²/sec E)NOTA

9. Find the length of the curve given by $9x^2 = 4y^3$ from $(0,0)$ to $(2\sqrt{3},3)$.

A) $\frac{10}{3}$ B) 4 C) $\frac{14}{3}$ D) $\frac{16}{3}$ E) NOTA

10. Find the volume of a solid with regular hexagonal cross-sections perpendicular to the x-axis and the longest diagonal of the hexagon lying in the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

- A) $12\sqrt{3}$ B) $24\sqrt{3}$ C) $32\sqrt{6}$ D) $48\sqrt{3}$ E) NOTA

11. Given $f(x) = 2x^3 + 4$, the value of c on the interval $[0, 2]$ such that $f'(c)$ equals the average rate of change on the interval is:

- A) $\sqrt{\frac{2}{3}}$ B) $\sqrt{\frac{4}{3}}$ C) $\sqrt{2}$ D) 2 E) NOTA

12. Find the volume of the solid generated when the region bounded by the curve $x^2 + y^2 - 8x + 14y + 56 = 0$ is revolved about the x-axis.

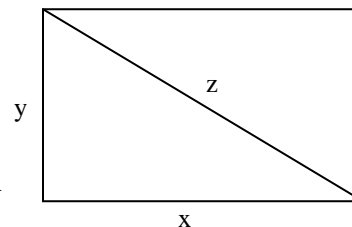
- A) $63\pi^2$ B) $72\pi^2$ C) $126\pi^2$ D) $144\pi^2$ E) NOTA

13. The sides of the rectangle shown increase in such a way that

$\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$ At the instant when $x = 4$ and

$y = 3$, what is the value of $\frac{dx}{dt}$

- A) $\frac{1}{3}$ B) 1 C) 2 D) $\sqrt{5}$ E) NOTA



14. A particle moves along the x-axis according to the equation $x(t) = t^3 - 3t^2 - 9t + 1$. Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

- A) 5 B) 6 C) 55 D) 59 E) NOTA

15. Water is leaking from my fish tank at the rate of $R(t) = \frac{12}{9 + 4t^2}$ gallons per minute, where t is the number of minutes since the leak began. How many gallons will have leaked out of the tank at $t = \frac{\sqrt{3}}{2}$?

- A) $\frac{\pi}{3}$ B) $\frac{\pi}{6}$ C) $\frac{5\pi}{6}$ D) $\frac{5\pi}{3}$ E) NOTA

22. Find the relation in terms of x given that $y' = xy^2$ and $y(3) = 2$

- A) $y = x^2 - 6$ B) $y = \frac{24 - x^2}{5}$ C) $y = \frac{x^2 - 5}{2}$ D) $y = \frac{2}{10 - x^2}$ E) NOTA

23. What is the average value of $y = x^2 \cdot \sqrt{x^3 + 1}$ on the interval $[0, 2]$

- A) $\frac{26}{9}$ B) $\frac{52}{9}$ C) $\frac{26}{3}$ D) $\frac{52}{3}$ E) NOTA

24. Find the area of one petal of the polar graph $r(\theta) = 4 \cos(2\theta)$.

- A) $\frac{\pi}{2}$ B) 2π C) 3π D) 4π E) NOTA

25. The amount of force required to push a cart up a small hill is directly proportional to the square of the distance from the base of the hill. It requires 40 Newtons of force to push the cart when it is 2 meters from the base of the hill. Find the work required to push the cart from a point that is 3 meters from the base of a hill to a point that is 6 meters from the base of the hill.

- A) 63 B) 270 C) 630 D) 720 E) NOTA

26. Find an equation for the line normal to $y = x^3 - 3x^2$ at its point of inflection.

- A) $y + 2 = -3(x - 1)$ B) $y - 2 = \frac{1}{3}(x + 1)$ C) $y + 1 = \frac{1}{3}(x - 2)$ D) $y + 2 = 3(x - 1)$ E) NOTA

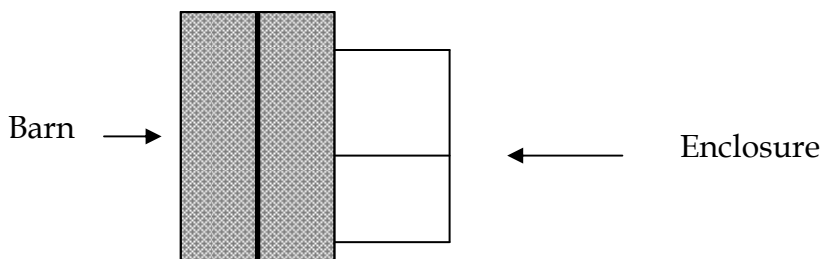
27. A particle moves clockwise around the ellipse $x^2 + 4y^2 = 20$. At the instant when it reaches the point $(2, 2)$ its x-coordinate is increasing at 4 units per second. Which of the following is true about the particle?

- A) its y-coordinate is also increasing at 4 units per second.
B) its y-coordinate is increasing at one unit per second.
C) its y-coordinate is decreasing at one unit per second.
D) its y-coordinate is decreasing at 4 units per second.
E) NOTA

28. A tank initially contains 40 gallons of water. Water pours into the tank at a rate of $3\sqrt{t}$ gallons per hour. How much water is in the tank four hours after the water starts pouring into the tank, where t is the time in hours after water starts flowing into the tank?

- A) 16 gal B) 46 gal C) 56 gal D) 64gal E) NOTA

29. 240 feet of fencing are going to be used to enclose a field next to a barn. Part of the fencing will be used to put a divider down the middle of the enclosure (see the diagram below). What are the dimensions of the enclosure with the largest possible area?



- A) 40ft by 120ft B) 60ft by 60ft C) 60ft by 120ft D) 30ft by 150ft
 E) NOTA

30. Mu Alpha Theta is selling candy bars to raise money for their trip to Rosemont. The quantity of candy bars they sell is defined by the equation $Q = 100e^{-0.1P}$, where P is the selling price, in cents per candy bar. How much should the group charge per bar in order to maximize revenue (Note: Revenue = price x quantity = $P \cdot Q$).

- A) \$1.00 B). \$100.00 C) \$0.50 cents
 D) The more they charge, the less they sell, so they should give them away.
 E) NOTA