

10 points for each question.

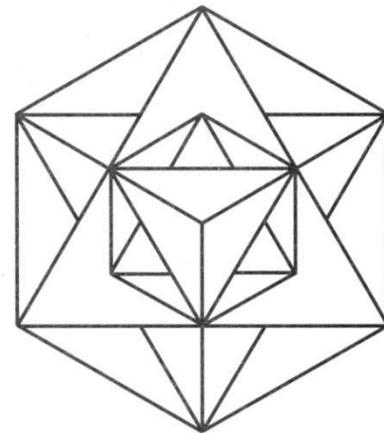
1. You are the ruler of a medieval empire and you are having a celebration tomorrow. The celebration is the most important party you have ever hosted. You have 1000 bottles of wine you were planning to open for the celebration, but you find out one of them is poisoned. The poison exhibits no symptoms until death. Death occurs within ten to twenty hours after consuming even the minutest amount of poison. You have over 1000 slaves at your disposal and just under 24 hours to determine which single bottle is poisoned. You have a handful of prisoners about to be executed, and it would mar your celebration to have anyone else killed. What is the smallest number of prisoners you must have to drink from the bottles to be absolutely sure to find the poisoned bottle within 24 hours?
2. A stark-raving mad king tells his 100 wisest men he is about to line them up and that he will place either a red or blue hat on each of their heads. Once lined up, they must not communicate amongst themselves, nor may they attempt to look behind them or remove their own hat. The king tells the wise men that they will be able to see all the hats in front of them. They will not be able to see the color of their own hat or the hats behind them, although they will be able to hear the answers from all those behind them. The king will then start with the wiseman in the back and ask, "What color is your hat?" The wise man will only be allowed to answer "red" or "blue". If the answer is incorrect, then the wise man will be silently killed. If the answer is correct, then the wise man may live but must remain absolutely silent. The king will then move on to the next wise man and repeat the question. The king makes it clear that if anyone breaks the rules then all the wise men will die. He then allows the wise men to consult before lining them up. The king listens in while the wise men consult each other to make sure they don't devise a plan to cheat. To communicate anything more than their guess of red or blue by coughing or shuffling would be breaking the rules. What is the maximum number of men they can be guaranteed to save?
3. You have twelve coins, one of which is known to be fake. The only thing distinguishing the fake coin from the real coins is that its weight is imperceptibly different. You have a perfectly balanced scale. The scale only tells you which side weights more than the other side. What is the smallest number of times you must use the scale in order to always find the fake coin? Use only the twelve coins themselves and no others, no other weights, no cutting coins, no pencil marks on the scale, etc. These are modern coins, so the fake coin is not necessarily lighter. Presume the worst-case scenario, and don't hope that you will pick the right coin on the first attempt.
4. What is the first four-digit number in which the first digit is one-third the second, the third is the sum of the first and second, and the last is three times the second?

5. Ten people land on a deserted island. There they find lots of coconuts and a monkey. During their first day they gather coconuts and put them all in a community pile. After working all day they decide to sleep and divide them into ten equal piles the next morning. That night one castaway wakes up hungry and decides to take his share early. After dividing up the coconuts he finds he is one coconut short of ten equal piles. He also notices the monkey holding one more coconut, so he tries to take the monkey's coconut to have a total evenly divisible by 10. However, when he tries to take it the monkey conks him on the head with it and kills him. Later another castaway wakes up hungry and decides to take his share early. On the way to the coconuts he finds the body of the first castaway, which pleases him because he will now be entitled to $\frac{1}{9}$ of the total pile. After dividing them up into nine piles he is again one coconut short and tries to take the monkey's slightly bloodied coconut. The monkey conks the second man on the head and kills him. One by one each of the remaining castaways goes through the same process until the 10th person to wake up gets the entire pile for himself. What is the smallest possible number of coconuts in the pile, not counting the monkey's coconut?
6. One morning it starts to snow at a constant rate. Later, at 6 am, a snow plow sets out to clear a straight street. The plow can remove a fixed volume of snow per unit time; i.e., its speed is inversely proportional to the depth of the snow. If the plow covered twice as much distance in the first hour as in the second hour, at what time, to the nearest minute, did it start snowing?
7. An ant and a blind spider are on opposite corners of a cube. The ant is stationary and the spider moves at random from one corner to another along the edges only. What is the expected number of turns before the spider reaches the ant?
8. The following verse spells out a word, letter by letter. What is the word?
- My first is in fish but not in snail
My second in rabbit but not in tail
My third in up but not down
My fourth in tiara not in crown
My fifth in tree you plainly see
My whole a food for you and me
9. If you add the age of a man to the age of his wife, the result is 91. He is now twice as old as she was when he was as old as she is now. How old is the man and his wife?
10. I live on Sunset Boulevard, where there are 6 houses on my side of the block. The house numbers are consecutive even numbers. The sum of all 6 house numbers is 8790. You don't know which block I live on, and it's a long street, but I will tell you that I live in the lowest number on my side of the block. What's my street address?

11. Mary had a coin purse with fifty coins totaling exactly \$1. Unfortunately, while counting her change, she dropped one coin. What is the probability that it was a penny?
12. Two perfect logicians, S and P, are told that integers x and y have been chosen such that $1 < x < y$ and $x + y < 100$. S is given the value of the sum of x and y while P is given the value of their product. They then have the following conversation:
- P: I cannot determine the two numbers.
S: I knew that.
P: Now I can determine them.
S: So can I.
- Given that the above statements are true, what are the two numbers?
13. Player A has one more coin than player B. Both players throw all of their coins simultaneously and observe the number that come up heads. Assuming all the coins are fair, what is the probability that A obtains more heads than B?
14. At a movie theater, the manager announces that a free ticket will be given to the first person in line whose birthday is the same as someone in line who has already bought a ticket. You have the option of getting in line at any time. Assuming that you don't know anyone else's birthday, and that birthdays are uniformly distributed throughout a 365-day year, what position in line gives you the best chance of being the first duplicate birthday?
15. Let $\triangle ABC$ be an isosceles triangle with $|\overline{AB}| = |\overline{AC}|$ and $\angle BAC = 20^\circ$. Point D is on side \overline{AC} such that $\angle DBC = 60^\circ$, and point E is on side \overline{AB} such that $\angle ECB = 50^\circ$. Find the measure of $\angle EDB$.
16. Using only the numbers 1, 3, 4, and 6 (each exactly once), together with the operations $+$, $-$, \div , and \times , and unlimited use of brackets, create an expression that equals 24. The operations may be used any number of times; decimal points are not allowed; and concatenating digits, such as $3 \times (14 - 6)$, is not allowed.
17. What digit did Arab mathematician al-Khwarizmi give to the West?
18. What three letters are each symbols for 1000?
19. What's short for "binary digit"?
20. In what century did mathematicians first use plus and minus signs?

21. What number, a one followed by 100 zeros, was first used by nine-year-old Milton Sirotta in 1940?
22. What handy mathematical instrument's days were numbered when the pocket calculator became big in the 1970s?
23. There is a very special five-digit number. The special thing about it is that its first digit describes how many zeros are digits in the whole number, the second digit describes how many ones are digits in the whole number, and so on, all the way to the fifth digit describing how many fours are digits in the whole number. What is the number?
24. The sum of four numbers is 100. If you add 4 to the first number, subtract 4 from the second number, multiply the third number by 4, and divide the fourth number by 4, then the four numbers will be the same. In order, what are the original four numbers?
25. You and your opponent alternate picking integers from 1 to 6 and a running total is kept. If the object of the game is to be the player who makes the total exactly 50, and you go first, what number should you pick to guarantee a win for yourself?

26. How many triangles are in the figure to the right?



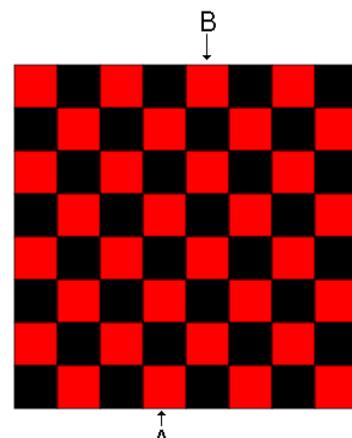
27. Every blip is a blop. Half of all blops are blips. Half of all bleeps are blops. There are 30 bleeps and 20 blips. No bleep is a blip. How many blops are neither blips nor bleeps?

28. There are ten positive integers less than 1000 that cannot be written as the sum of at least two consecutive natural numbers. For example, $9 = 4 + 5$. and $10 = 1 + 2 + 3 + 4$. What is the sum of all ten of these numbers?

29. On the checkerboard to the right, how many different ways are there to move a checker from position A to position B so it can be made a king?

30. If you were to construct a 7×7 checkered square, how many rectangles would there be on it in total?

31. If measured from the center of the planet, which mountain on Earth is the highest?

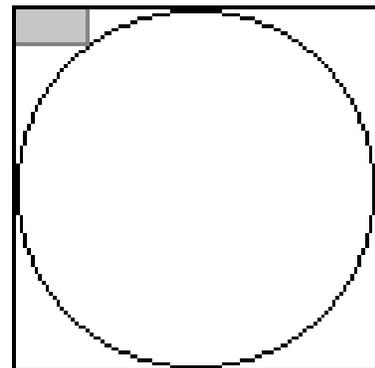


32. My grandson is about as many days old as my son is weeks old, and my grandson is as many months old as I am years old. My grandson, my son, and I together are 140 years old. How old am I, in years? For this problem, assume 52 weeks in a year and 365 days in a year.
33. What is the only English word (up to variation) with three consecutive repeated letters? For example, the word "sweet-toothed" would be one if it weren't for the hyphen.
34. You are to open a safe without knowing the combination. Beginning with the dial set at 0, the dial must be turned counter-clockwise to the first combination number (then clockwise back to 0), clockwise to the second combination number (then counter-clockwise back to 0), and counter-clockwise to the third combination number, whereupon the door shall immediately spring open. There are 40 numbers on the dial, including the 0. Without knowing the combination numbers, what is the maximum number of trials required to open the safe (one trial is one attempt to dial a full three-number combination)?
35. If written in binary, in how many consecutive zeros would the base-10 number $100!$ end?
36. On an early morning, three rivals get together to settle a quarrel by means of a duel: a three-person duel with people A, B, and C. The rules of the duel are as follows:
1. The three draw lots to see who many fire first, second, and third.
 2. The three people fire at each other in this order until only a single person is alive.
 3. Each person decides for himself at which person he will fire.
 4. All three know that A is a 100% kill-shot, B is an 80% kill-shot, and C is a 50% kill-shot.
 5. Each person choose his own ideal strategy.
 6. No one is killed by a stray bullet.

What is the probability that C will survive?

37. A man gives each of his seven children an allowance of one dollar. Each child gets the same number of coins (using only quarters, dime, nickels, and pennies), but no two children get the same combination of coins. What is the least number of coins he needs to pay all of the allowances?

38. In the figure to the right, the rectangle in the upper left-hand corner is has two sides that lie on the sides of the square and whose lower right-hand corner lies on the circle. If that smaller rectangle measures 3 cm by 6 cm, what is the radius of the circle, in cm?



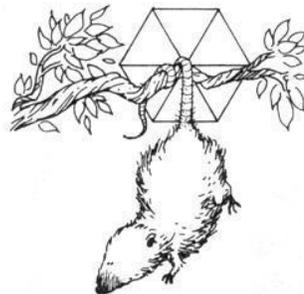
39. Divide \$1040 (in whole-dollar increments) into a number of bags so that I can ask for any amount between \$1 and \$1040, inclusive, and you can give me that amount by giving me a certain number of these bags. What is the smallest number of bags needed to do this?
40. There are 100 light bulbs lined up in a row in a long room. Each bulb has its own switch and is currently switched off. The room has an entry door and an exit door. There are 100 people lined up outside the entry door. Each bulb is numbered consecutively from 1 to 100, as is each person. Person 1 enters the room, switches on every bulb, then exits. Person 2 enters and flips the switch on every second bulb (turning off bulbs 2, 4, 6,...). Person 3 enters and flips the switch on every third bulb. This continues until all 100 people have passed through the room. How many light bulbs are illuminated after the 100th person has passed through the room?
41. At the beginning of the first World War, the uniform of the British soldiers included a brown cloth cap. They were not provided with metal helmets. As the war went on, the army authorities and the War Office became alarmed at the high proportion of men suffering head injuries. They therefore decided to replace the cloth headgear with metal helmets. From then on, all soldiers wore the metal helmets. However, the War Office was amazed to discover that the incidence of head injuries then increased. It can be assumed that the intensity of fighting was the same before and after this change. So why should the recorded number of head injuries per battalion increase when men wore metal helmets rather than cloth caps?
42. There are two glasses on a table: one containing water and the other containing milk. They both contain exactly the same amount by volume. If you take a teaspoon of water and mix it into the milk, then take a teaspoon from the milk glass and mix it with the water, both glasses become contaminated. Which glass is more contaminated?
43. Farmer Giles has four sheep. One day he notices that they are standing in such a way that they are all the same distance away from each other; i.e., the distance between any two of the four sheep is the same. How can this be so?
44. An ancient Arabic puzzle goes like this: A hunter met two shepherds, one of whom had three loaves of bread, the other five loaves. All the loaves were the same size. The three men agreed to share the eight loaves equally between them. After they had eaten, the hunter gave the shepherds eight bronze coins as payment for his meal. How should the two shepherds fairly divide this money?
45. 8,549,176,320 is a one-of-a-kind number. What is so special about it?
46. Find all five-digit numbers whose digits are reversed when multiplied by 4.

47. During a grim medieval battle, 85% of the warriors lost an ear, 80% lost an eye, 75% lost an arm, and 70% lost a leg. What is the smallest percentage possible of combatants who lost all of the above?
48. Given that one and only one answer is correct, which of the following is true?
- All of the below
 - None of the below
 - One of the above
 - All of the above
 - None of the above
 - None of the above
49. Which weighs more: a pound of gold or a pound of lead?
50. Use each digit 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 exactly once to form two five-digit numbers that when multiplied produce the largest quantity. What is the value of this product?
51. My son learned how to count in a different base than 10, so that, for instance, instead of writing 136, he writes 253. In what base does he count?
52. How far is the horizon from the top of a 125.7-meter-high lighthouse? For this problem, consider the earth a sphere with circumference of its great circle of 40,000 km.
53. Knowing that the probability that at least two of us have birthdays on the same day is less than half, but that this would not be the case were we one more in number, how many people are there?
54. Using all the integers from 1 to 9, create a single fraction that equals exactly $\frac{1}{3}$. Each number may only be used once.

55.



56.



57.



58.

