Answers:

1. E
2. B
3. D
4. D
5. A
6. B
7. D
8. B
9. A
10. B
11. B
12. B
13. A
14. B
15. C
16. D
17. C
18. A
19. B
20. A
21. D
22. A
23. C
24. B
25. A
26. C
27. C
28. B
29. C
30. D

Solutions:

1. $\dfrac{10^{2009}+10^{2011}}{10^{2010}+10^{2010}}=\dfrac{10^{2009}\left(1+10^2\right)}{10^{2009}\left(10+10\right)}=\dfrac{101}{20}$

2. Beginning with $(5!)^2$, all numbers in the summation end in a 0. So we only need to find the unit's digit of the sum of the first four terms. $(1!)^2+(2!)^2+(3!)^2+(4!)^2$ $=1+4+36+576=617$, so the unit's digit is a 7.

3. 64 can be written as 2^6, 4^3, 8^2, or 64^1 when written as powers of positive integers, so the number of ordered triples is the number of ways of writing the exponents as an ordered product of positive integers. $6=1\cdot6=2\cdot3=3\cdot2=6\cdot1$ (4 ways), $3=1\cdot3$ $=3\cdot1$ (2 ways), $2=1\cdot2=2\cdot1$ (2 ways), and $1=1\cdot1$ (1 way), so there are 9 total ways of writing the solutions as ordered triples.

4. $\dfrac{4}{A^2}=\dfrac{9B}{A}-2B^2\Rightarrow4=9BA-2B^2A^2\Rightarrow0=2(AB)^2-9AB+4=(2AB-1)(AB-4)$
$\Rightarrow AB=\dfrac{1}{2}$ or $AB=4\Rightarrow A=\dfrac{1}{2}B^{-1}$ or $A=4B^{-1}$. $\dfrac{1}{2}-1+4-1=\dfrac{5}{2}$

5. $k^{(\log_2 5)^2}=\left(k^{\log_2 5}\right)^{\log_2 5}=16^{\log_2 5}=5^{\log_2 16}=5^4=625$

6. $\left(x+\dfrac{1}{x}\right)^2=x^2+\dfrac{1}{x^2}+2=\dfrac{6}{7}+\dfrac{7}{6}+2=\dfrac{36+49+84}{42}=\dfrac{169}{42}$

7. $A=\dfrac{(\log_3 1-\log_3 4)(\log_3 9-\log_3 2)}{(\log_3 1-\log_3 9)(\log_3 8-\log_3 4)}=\dfrac{(-\log_3 4)\left(\log_3\dfrac{9}{2}\right)}{(-\log_3 9)(\log_3 2)}=(\log_9 4)\left(\log_2\dfrac{9}{2}\right)$
$=(\log_3 2)\left(\log_2\dfrac{9}{2}\right)=\log_3\dfrac{9}{2}\Rightarrow3^A=3^{\log_3\frac{9}{2}}=\dfrac{9}{2}$

8. $3+3\log_3\left(x^3+1\right)=3^2\Rightarrow3\log_3\left(x^3+1\right)=6\Rightarrow\log_3\left(x^3+1\right)=2\Rightarrow9=x^3+1\Rightarrow x^3=8$
$\Rightarrow x=2$

9. Let $A=2011$. So $\sqrt{2014\cdot2012\cdot2010\cdot2008+16}=\sqrt{(A+3)(A+1)(A-1)(A-3)+16}$
$=\sqrt{\left(A^2-9\right)\left(A^2-1\right)+16}=\sqrt{A^4-10A^2+25}=A^2-5=4{,}044{,}116$

10. $e^2-5e+6=(e-2)(e-3)\approx(.718)(-.282)=-.202476$

11. $2^{2009}\cdot 5^{2011}=10^{2009}\cdot 25$, so the sum of the digits is $2+5=7$

12. The unit's digit will be 3 if the power of 7 is 1 less than a multiple of 4. Therefore, if the power is 3, 7, 11, …, 2011, the unit's digit will be 3. These are the powers all 1 less than $1\cdot 4,\ 2\cdot 4,\ \ldots,\ 503\cdot 4$, so there are 503 terms that fit the criterion.

13. Let x_1 and x_2 be the solutions to $x^2-cx+d=0$. Then $x_1+x_2=c$ and $x_1x_2=d$. Also, $a=x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=c^2-2d$.

14. $x=\sqrt{5+\sqrt{5+\sqrt{5+\ldots}}}\Rightarrow x=\sqrt{5+x}\Rightarrow x^2=5+x\Rightarrow x^2-x-5=0\Rightarrow x=\dfrac{1\pm\sqrt{1+20}}{2}$

$=\dfrac{1\pm\sqrt{21}}{2}$, but x must be positive, so $x=\dfrac{1+\sqrt{21}}{2}$. $y=\dfrac{5}{1+\dfrac{5}{1+\dfrac{5}{1+\ldots}}}\Rightarrow y=\dfrac{5}{1+y}$

$\Rightarrow y^2+y-5=0\Rightarrow y=\dfrac{-1\pm\sqrt{1+20}}{2}=\dfrac{-1\pm\sqrt{21}}{2}$, but y must be positive, so

$y=\dfrac{-1+\sqrt{21}}{2}$. $x+y=\dfrac{1+\sqrt{21}}{2}+\dfrac{-1+\sqrt{21}}{2}=\dfrac{2\sqrt{21}}{2}=\sqrt{21}$

15. The sum is $(1-i)+(1-i)^3+\ldots+(1-i)^{13}+(1+i)^2+(1+i)^4+\ldots+(1+i)^{14}$

$=\dfrac{(1-i)\left(1-(1-i)^{14}\right)}{1-(1-i)^2}+\dfrac{(1+i)^2\left(1-(1+i)^{14}\right)}{1-(1+i)^2}=\dfrac{(1-i)(1-128i)}{1+2i}+\dfrac{(2i)(1+128i)}{1-2i}$

$=\dfrac{(-127-129i)(1-2i)+(-256+2i)(1+2i)}{5}=\dfrac{-385+125i-260-510i}{5}=-129-77i$

16. To be defined, we must have either $x<0$ and $x^2-1<0$ OR $x>0$ and $x^2-1>0$. Therefore, x must be in the intervals $(-1,0)\cup(1,\infty)$.

17. $x=2^{\left(3^{\left(4^1\right)}\right)}=2^{81}=10^{81\log 2}=10^{81(.301)}=10^{24.381}$, which equals $c\cdot 10^{24}$ for some number c, $1\le c<10$. Therefore, this number has 25 digits.

18. $x=\log_5\left(3f^{-1}(x)\right)-1\Rightarrow x+1=\log_5\left(3f^{-1}(x)\right)\Rightarrow 3f^{-1}(x)=5^{x+1}=5\cdot 5^x$

$\Rightarrow f^{-1}(x)=\frac{5}{3}(5^x)$

19. $160=10r^8 \Rightarrow r^8=16 \Rightarrow r=\sqrt{2}$, so one hour later the population is $160\sqrt{2}$ $\approx 160(1.414)=226.24$, which is closest to 226.

20. $x^2=x+6 \Rightarrow 0=x^2-x-6=(x-3)(x+2) \Rightarrow x=3$ or $x=-2$, and both expressions are defined for both values, so the sum of the y-values is $\log_6(3)^2+\log_6(-2)^2$ $=\log_6 9+\log_6 4=\log_6 36=2$.

21. The slope between the points is $\frac{185/9-5/9}{173/3+7/3}=\frac{20}{60}=\frac{1}{3}$, so the equation of the segment is $y-\frac{5}{9}=\frac{1}{3}\left(x+\frac{7}{3}\right) \Rightarrow y=\frac{1}{3}x+\frac{4}{3} \Rightarrow 3y=x+4$. The smallest x-value that works would be -1, which occurs when $y=1$, meaning the point is $(-1,1)$. To find the next point, add 3 to the x-coordinate and 1 to the y-coordinate. So the next point is $(2,2)$, the next point is $(5,3)$, and this pattern continues until the final point, $(56,20)$, is reached, which is a total of 20 lattice points. If $\log_2(20-K)$ is an integer, the largest value of $20-K$ would be 16, which would yield the smallest positive integer value of K, which would be 4.

22. The sought term is $\binom{6}{2}(a^2b^{-1})^4\left(-b^{1/2}a^{-3}\right)^2=15(a^8b^{-4})(ba^{-6})=15a^2b^{-3}$, so the coefficient is 15.

23. Multiplying the equation by $\log_x 3$ gives $(\log_x 3)^2+1=3\log_x 3$ $\Rightarrow(\log_x 3)^2-3\log_x 3+1=0$. Using the quadratic formula, $\log_x 3=\frac{3\pm\sqrt{9-4}}{2}=\frac{3\pm\sqrt{5}}{2}$. Therefore, the largest value of $\log_x 3$ is $\frac{3+\sqrt{5}}{2}$.

24. $\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{6}+\sqrt{2}}{4}i\right)^{2011}=(cis75^\circ)^{2011}=(cis1800^\circ)^{83}(cis75^\circ)^{19}=cis1425^\circ$ $=cis345^\circ=\frac{\sqrt{6}+\sqrt{2}}{4}-\frac{\sqrt{6}-\sqrt{2}}{4}i$

25. $\sqrt{\sum_{n=0}^{2}\left(n+\sqrt{2}\right)^2}=\sqrt{2+\left(3+2\sqrt{2}\right)+\left(6+4\sqrt{2}\right)}=\sqrt{11+6\sqrt{2}}=3+\sqrt{2}$

26. $x=\sqrt{8+\dfrac{8}{\sqrt{8+\dfrac{8}{\sqrt{8+\dfrac{8}{\dots}}}}}}\Rightarrow x=\sqrt{8+\dfrac{8}{x}}\Rightarrow x^2=8+\dfrac{8}{x}\Rightarrow x^3=8x+8\Rightarrow 0=x^3-8x-8$

$=(x+2)\left(x^2-2x-4\right)$, so $x=-2$ or $x=\dfrac{2\pm\sqrt{4+16}}{2}=\dfrac{2\pm 2\sqrt{5}}{2}=1\pm\sqrt{5}$. Since x must be positive, we must have $x=1+\sqrt{5}$.

27. This sequence of terms is 1, 2, 2, 3, 3, 3, …, where n n's appear in the list. Therefore, the last n is the $\dfrac{n(n+1)}{2}$st term in the sequence. The last 62 is the 1953rd term, so the sum is $1^2+2^2+3^2+\dots+62^2+58\cdot 63=\dfrac{62\cdot 63\cdot 125}{6}+3654=81375+3654=85029$.

28. Let $A=\left(\sqrt{3}+\sqrt{2}\right)^6$ and $B=\left(\sqrt{3}-\sqrt{2}\right)^6$. Then $A+B$

$=2\left(\sqrt{3}^6+15\sqrt{3}^4\sqrt{2}^2+15\sqrt{3}^2\sqrt{2}^4+\sqrt{2}^6\right)=2(27+270+180+8)=970$, and

$0<B\approx(1.732-1.414)^6<\left(1/2\right)^6=1/64$, so $A+B$ is closest to 970.

29. The equation is true as long as x is a number that makes all three logarithms defined. For the first one, $x>0$. For the second one, $x>2$. For the third one, $x<0$ or $x>2$. Therefore, all three logarithms are defined for $x>2$.

30. $0=3^{4x}-3^{2x+\log_3 12}+27=\left(3^{2x}\right)^2-12\left(3^{2x}\right)+27=\left(3^{2x}-3\right)\left(3^{2x}-9\right)\Rightarrow 3^{2x}=3$ or $3^{2x}=9$

$\Rightarrow 2x=1$ or $2x=2\Rightarrow x=1/2$ or $x=1$, so the sum of the solutions is $1/2+1=3/2$.