

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. Evaluate: $(97 + 87 + 77 + 67) - (7 + 17 + 27 + 37)$

- A) 200 B) 240 C) 300 D) 360 E) NOTA

2. How many terms are in the arithmetic sequence $-8, -5, -2, \dots, 2011$?

- A) 671 B) 672 C) 673 D) 674 E) NOTA

3. Find the prime factorization of the 10th term of the geometric series with first term 6 and common ratio 12.

- A) $2^{19} \cdot 3^{10}$ B) $2^{20} \cdot 3^{10}$ C) $2^{21} \cdot 3^{11}$ D) $2^{22} \cdot 3^{11}$ E) NOTA

4. The first term of an arithmetic sequence is 2011 and the 2011th term is 1. Find the 100th term of the sequence.

- A) 100 B) 1776 C) 1912 D) 2110 E) NOTA

5. The second term of a geometric sequence is 144 and the fourth term is 324. Find the sum of all possible values of the first term of the sequence.

- A) 0 B) -96 C) 216 D) -216 E) NOTA

6. Let $t_1 = 2$, and let $t_{n+1} = 3t_n + 2$ for all integers $n \geq 1$. Find the base-3 representation of t_6 .

- A) 121212 B) 222222 C) 1000002 D) 102102 E) NOTA

7. Find the positive integer n such that $15 + 16 + 17 + \dots + n = 15n$

- A) -6 B) 21 C) 35 D) 85 E) NOTA

8. Compute the sum of the multiples of 9 between 100 and 1000.

- A) 1107 B) 45,450 C) 54,900 D) 55,350 E) NOTA

9. Evaluate: $\sum_{n=7}^{17} (7n + 17)$

- A) 264 B) 999 C) 1010 D) 1111 E) NOTA

10. A repunit is a positive integer whose digits are all equal to 1. For instance, 1, 11, 111, and 1111 are the four smallest repunits. If $k = 39^2 - 4$, find the number of digits in the smallest repunit that is divisible by k .

- A) 6 B) 8 C) 15 D) 120 E) NOTA

11. Note that $\binom{0}{0} = 1$, $\binom{1}{0} = 1$, $\binom{2}{0} + \binom{1}{1} = 2$, $\binom{3}{0} + \binom{2}{1} = 3$, $\binom{4}{0} + \binom{3}{1} + \binom{2}{2} = 5$, and $\binom{5}{0} + \binom{4}{1} + \binom{3}{2} = 8$. Compute $\binom{6}{0} + \binom{5}{1} + \binom{4}{2} + \binom{3}{3}$.

- A) 12 B) 13 C) 14 D) 21 E) NOTA

12. Anshu writes down three numbers that are in an arithmetic progression. He notices that the sum of the three numbers is 21 and that the product of the two larger numbers is exactly twice the product of the two smaller numbers. What is the largest of Anshu's three numbers?

- A) 8 B) $3\frac{3}{4}$ C) $7 + \sqrt{2}$ D) $2\frac{8}{3}$ E) NOTA

13. Let a , b , and c be real numbers that satisfy the system of equation $\begin{cases} a + b + 2c = 4 \\ a + 2b + 4c = 7 \\ 2a + 4b + 7c = 13 \end{cases}$.

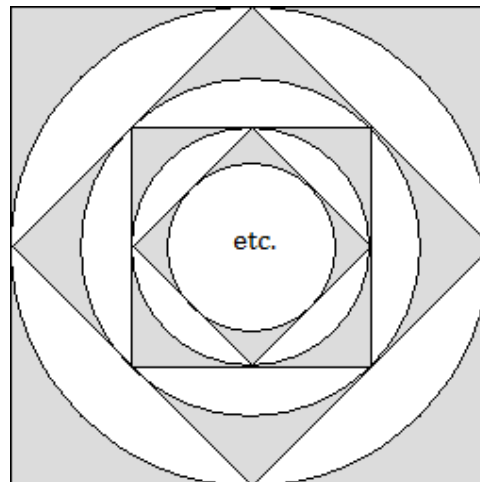
Compute the value of $7a + 13b + 24c$.

- A) 24 B) 41 C) 44 D) 81 E) NOTA

14. Sameera plays a challenging game in which she achieves integer scores from one to ten points each game. She scores one point ten times, two points nine times, three points eight times, four points seven times, five points six times, six points five times, seven points four times, eight points three times, nine points two times, and one time she scores a perfect score of ten. What is the mean number of points Sameera scores among all the times she plays the game?

- A) 3 B) $4\frac{0}{11}$ C) $1\frac{8}{5}$ D) 4 E) NOTA

15. In the diagram, one circle is inscribed in each square and one square is inscribed in each circle, with this process being continued infinitely. The area between each square and the circle inscribed in it is shaded. What proportion of the area inside the largest square is shaded?



- A) $4 - \pi$ B) $\frac{1}{2} + \frac{\pi}{16}$ C) $\frac{1}{2} + \frac{\pi}{8}$
 D) $2 - \frac{\pi}{2}$ E) NOTA
16. The sum of the first three terms of an arithmetic sequence is -300 and the sum of the first nine terms of the same sequence is 300 . Find the sum of the first six terms of the arithmetic sequence.
- A) -200 B) -100 C) 0 D) 400 E) NOTA
17. Let b_1, b_2, \dots be a geometric sequence such that $b_1 + b_2 = 1$ and $\sum_{k=1}^{\infty} b_k = 2$. Given that $b_2 < 0$, compute the value of b_1 .
- A) $2 - \sqrt{2}$ B) $1 + \sqrt{2}$ C) $\frac{\sqrt{5} + 1}{2}$ D) $4 - \sqrt{5}$ E) NOTA
18. Let $p(x)$ be a cubic polynomial with $p(0) = 1$, $p(2) = 15$, $p(4) = 133$, and $p(6) = 427$. Determine the value of $p(-2)$.
- A) 19 B) 23 C) 965 D) 969 E) NOTA
19. Define $a_k = 2^k$, $b_k = \sum_{n=1}^k a_n$, and $c_k = \sum_{m=1}^k b_m$ for all positive integers k . Compute c_{10} .
- A) 2024 B) 2035 C) 4072 D) 4083 E) NOTA
20. The first term of a geometric sequence is greater than 10 and the fifth term is less than 1000. If r is the common ratio and real, how many integers could be equal to r ?
- A) 3 B) 4 C) 6 D) 7 E) NOTA

21. Let R be the set of remainders possible when a positive integer is divided by 8. Let T be the subset of R of remainders possible when a Fibonacci number is divided by 8. Find the sum of the elements of T .

- A) 10 B) 18 C) 22 D) 24 E) NOTA

22. Consider the sequence $a_1 = 1, a_2 = 13, a_3 = 135, \dots$ of positive integers. The k th term a_k is defined by appending the digits of the k th smallest positive odd integer to the preceding term a_{k-1} . Note that $a_4 = 1357, a_5 = 13579$, and $a_6 = 1357911$. Find the value of n such that a_n is the 100th term of the sequence that is divisible by 3.

- A) 200 B) 300 C) 333 D) 500 E) NOTA

23. Let $S = \sum_{k=1}^8 \log_5 k$. Compute the value of S , rounded to the nearest integer.

- A) 5 B) 6 C) 7 D) 8 E) NOTA

24. Compute the value of $\left\lfloor \frac{2011!}{2010! + 2009! + 2008! + \dots + 1!} \right\rfloor$, where $\lfloor x \rfloor$ represents the greatest integer less than or equal to x .

- A) 2007 B) 2008 C) 2009 D) 2010 E) NOTA

25. Let a_n be the n th term of a sequence, and define $S_n = \sum_{k=1}^n a_k$. If $S_n = 4n^2 + 2n + 8$, find the value of a_{428} .

- A) 428 B) 855 C) 1710 D) 3422 E) NOTA

26. Meena has a bouncy ball that always rebounds to half the vertical distance from which it falls. She drops it out of the window of her dorm room from a height of 32 feet. After a couple of bounces, the ball bounces down a set of 10 steps that each drop 2 feet. It hits exactly three of the steps as it goes down, never hitting the corner of a step so that it continues bouncing, rebounding half the vertical distance it drops on each bounce. The ball eventually comes to rest at the bottom of the stairs. Find the total vertical distance, in feet, the ball travels before coming to rest.

- A) 156 B) 182 C) 204 D) 260 E) NOTA

27. An infinite geometric series has a positive common ratio and third term equal to 8.

Given that this infinite geometric series converges, find the smallest possible value of its sum.

- A) 54 B) 56 C) 64 D) 72 E) NOTA

28. A Pythagorean triple (a, b, c) , not necessarily primitive, is such that $a < b < c$ and

$b, 2a, c$, in that order, form an arithmetic progression. How many possible values of c are less than 100?

- A) 3 B) 4 C) 5 D) 6 E) NOTA

29. A lemur starts at the origin of the Cartesian coordinate plane and moves 1 unit along

the positive x -axis to the point $(1, 0)$, which is referred to as node 1. The lemur then

turns 90° counterclockwise and moves 2 units to the point $(1, 2)$, which is referred to as

node 2. The lemur continues to turn 90° counterclockwise at each node and moves one

unit farther than it moved before, always moving n units to get to node n . At node 100,

the lemur stops. At how many of the nodes (from 1 to 100, inclusive) was the lemur an integer distance from the origin?

- A) 1 B) 2 C) 3 D) 4 E) NOTA

30. Simplify: $2\sin 2^\circ + 4\sin 4^\circ + 6\sin 6^\circ + \dots + 2n\sin(2n)^\circ + \dots + 180\sin 180^\circ$

- A) $45\sin 89^\circ$ B) $45\tan 89^\circ$ C) $90\cos 1^\circ$ D) $90\cot 1^\circ$ E) NOTA