

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. What is the sum of the first 1000 positive integers?

- A) 50,500 B) 500,000 C) 500,500 D) 1,001,000 E) NOTA

2. What is the sum of the integers between 100 and 199, inclusive?

- A) 14,950 B) 15,000 C) 15,050 D) 15,100 E) NOTA

3. Evaluate: $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots + \frac{(-1)^{n+1}}{3^{n-1}} + \dots$

- A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) $\frac{8}{9}$ D) $\frac{4}{3}$ E) NOTA

4. Evaluate: $\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin n^\circ + \dots + \sin 360^\circ$

- A) -90 B) 0 C) 45 D) $45\sqrt{2}$ E) NOTA

5. Consider the sequence formed by the digits of the concatenation of the positive integers, listed in order: 123456789101112131415161718192021... What is the 2011th digit of this sequence?

- A) 0 B) 1 C) 6 D) 7 E) NOTA

6. Let s be the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. Which one of the following open intervals contains s ?

- A) $(\frac{1}{12}, \frac{1}{3})$ B) $(\frac{1}{3}, \frac{7}{12})$ C) $(\frac{7}{12}, \frac{5}{6})$ D) $(\frac{5}{6}, \frac{13}{12})$ E) NOTA

7. Evaluate the double sum: $\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{1}{n^m}$

- A) 1 B) 2 C) 4 D) e E) NOTA

8. If $i = \sqrt{-1}$, find the value of the sum $i + i^2 + i^3 + i^4 + \dots + i^n + \dots + i^{2011}$.

- A) -1 B) $-i$ C) $-1+i$ D) $1+i$ E) NOTA

9. Steven takes an arithmetic sequence with common difference d and a geometric sequence with common ratio r and adds them together, term by term, to form a new sequence (i.e., the first term of the new sequence is the sum of the first terms of the arithmetic and geometric sequences, etc.). If the first three terms of the new sequence are, in order, 3, 8, and 15, and d and r are both known to be positive integers, what is the sum of the possible values of d ?

- A) 3 B) 7 C) 20 D) 25 E) NOTA

10. Define a sequence a_n by $a_0 = 1$ and $a_{n+1} = \frac{1}{1+a_n}$ for all integers $n \geq 1$. What value do the terms of this sequence approach as n becomes large without bound? In other words,

find the value of the continued fraction $\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$.

- A) 0 B) 1 C) $\frac{1+\sqrt{5}}{2}$ D) $\frac{-1+\sqrt{5}}{2}$ E) NOTA

11. First discovered by Euler, the exact value of the sum of the reciprocals of the perfect squares, $\sum_{i=1}^{\infty} \frac{1}{i^2}$, is $\frac{\pi^2}{6}$. What is the value of $\sum_{i=1}^{\infty} \frac{1}{(2i-1)^2}$, which is the sum of the reciprocals of the odd perfect squares?

- A) $\frac{\pi^2}{12}$ B) $\frac{\pi^2}{9}$ C) $\frac{\pi^2}{8}$ D) $\frac{\pi^2-2}{6}$ E) NOTA

12. Define a sequence a_n by $a_0 = 0$, $a_1 = 1$, and $a_{n+1} = a_n + \frac{a_n - a_{n-1}}{2}$ for all integers $n \geq 1$.

What value do the terms of this sequence approach as n becomes large without bound?

- A) $\frac{2}{3}$ B) 4 C) 8 D) the sequence does not approach any number E) NOTA

13. If $i = \sqrt{-1}$, find the value of the sum $i \log 1 + i^2 \log 2 + \dots + i^n \log n + \dots + i^{10} \log 10$.

- A) $-1-i$ B) $-\log 3840 + i \log 755$ C) $\log\left(\frac{4}{15}\right) + i \log\left(\frac{15}{7}\right)$ D) $-\log(6) + i \log(4)$
E) NOTA

14. Define $S_n = \sum_{i=1}^n ((-1)^{i+1} i)$. Find the value of S_{2011} .

- A) -1 B) 1 C) -1005 D) 1006 E) NOTA

15. Simplify the following product: $\prod_{n=2}^{2011} \left(\sum_{i=0}^{\infty} \frac{1}{n^i} \right)$

- A) 2010 B) 2011 C) 2010! D) 2011! E) NOTA

16. A partition of a positive integer n is a way of writing n as the sum of one or more positive integers written in non-increasing order. For example, all partitions of the number 4 are 4, 3+1, 2+2, 2+1+1, and 1+1+1+1. How many partitions of 2011 express 2011 as the sum of exactly 2007 positive integers?

- A) 4 B) 5 C) 35 D) $\binom{2010}{4}$ E) NOTA

17. Three of the first five terms of an increasing arithmetic sequence are 5, 11, and 13. What is the 20th term of the sequence?

- A) 38 B) 41 C) 43 D) 45 E) NOTA

18. A business needs to hire three people to enter data into a database. Applicants must take a short exam, and the business immediately hires the first three people who meet a predetermined minimum score on the exam. If it takes a total of eight applicants until all three spots are filled, how many possible sequences are there for the way the applicants passed or failed the exam?

- A) 21 B) 42 C) 128 D) 256 E) NOTA

19. Consider the function $\Lambda(n) = \begin{cases} \ln p, & \text{if } n = p^k \text{ for some prime } p \text{ and positive integer } k \\ 0, & \text{otherwise} \end{cases}$. As

examples, $\Lambda(26) = 0$ and $\Lambda(169) = \Lambda(13^2) = \ln 13$. Find the sum of the series $\sum_{i|5400} \Lambda(i)$,

where i takes on only the positive integers which divide evenly into 5400.

- A) $\ln 30$ B) $\ln 5400$ C) $\ln 14582700$ D) $\ln(5400!)$ E) NOTA

20. Suppose that to each positive integer n we assign a unique, infinite sequence whose terms consist only of 0's and 1's. For example, the following table illustrates a possible choice of sequences:

Sequence assigned to 1	1,0,1,0,1,0,1,0,1,...
Sequence assigned to 2	0,1,0,0,1,0,0,0,1,...
Sequence assigned to 3	1,1,0,1,0,0,0,1,0,...
...	...

What infinite sequence consisting only of 0's and 1's can be guaranteed to differ from every one of this infinite number of infinite sequences we have chosen?

- A) the one whose n th term is the same as the n th term of the sequence assigned to n
- B) the one whose n th term is the opposite of the n th term of the sequence assigned to n
- C) the one whose n th term is the same as the first term of the sequence assigned to n
- D) no such sequence is guaranteed to exist
- E) NOTA

21. Evaluate: $\sum_{n=1}^{\infty} \frac{2n+1}{3^n}$

- A) 1.5
- B) 1.75
- C) 2
- D) 2.25
- E) NOTA

22. The first three terms of a quadratic sequence are 1, 3, and 1. Find the 4th term in the sequence. A quadratic sequence is a sequence whose n th term is given by $an^2 + bn + c$ for fixed constants a , b , and c .

- A) -7
- B) -1
- C) 0
- D) 3
- E) NOTA

23. Aaya counts to 100 in a strange way. She counts 1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, ..., starting over after each time she reaches a number she has not previously said. She does this until she counts 100 for the first time. How many numbers, including repetitions, does Aaya actually count?

- A) 1000
- B) 4950
- C) 5000
- D) 5050
- E) NOTA

24. A sequence has first term 1, and each successive term in the sequence is the sum of all the previous terms. What is the 2011th term in this sequence?

- A) 2^{2012}
- B) 2^{2010}
- C) 2^{2011}
- D) 2^{2009}
- E) NOTA

25. A sequence of positive integers starting with 2011 satisfies the following conditions:
 1) the terms alternative between odd and even; 2) no integer appears twice; and 3) the sum of any two consecutive terms is a multiple of 3. What is the sum of the four smallest positive integers that could never belong to such a sequence?

- A) 10 B) 18 C) 24 D) 30 E) NOTA

26. A 0-dimensional mouse scurries 4 meters ahead, turns left and scurries 2 meters ahead, then turns left again and scurries 1 meter ahead, and so on, always turning to its left and scurrying half the distance it just previously scurried. If the mouse continued this pattern indefinitely, how far away from where it started would the mouse ultimately wind up? Assume the mouse makes 90° turns.

- A) 8 meters B) 4.75 meters C) $\frac{8\sqrt{5}}{5}$ meters D) $\frac{3\sqrt{5}}{2}$ meters E) NOTA

27. If $i = \sqrt{-1}$, find the value of the sum $1 + (1-i) + (1-i)^2 + \dots + (1-i)^{n-1} + \dots + (1-i)^{10}$.

- A) $32 - 33i$ B) $-32 + i$ C) $-32 + 32i$ D) $-i$ E) NOTA

28. The product of three consecutive terms in a geometric sequence of real terms is 27. What is the value of the middle of these three terms?

- A) $\frac{27}{8}$ B) 3 C) 9 D) 27 E) NOTA

29. Approximate $1 + \frac{21}{41} + \left(\frac{21}{41}\right)^2 + \dots + \left(\frac{21}{41}\right)^{n-1} + \dots + \left(\frac{21}{41}\right)^{2011}$ to the nearest thousandth.

- A) 1.661 B) 1.892 C) 2.011 D) 2.050 E) NOTA

30. Let z_n , the n th term of a sequence of complex numbers, be defined by $z_n = (5 + 12i)\left(\frac{i}{2}\right)^n$. Let S be the intersection of the closed unit disk centered at the origin and the fourth quadrant in the complex plane. What is the smallest value of n such that z_n lies in S ?

- A) 4 B) 5 C) 6 D) 7 E) NOTA

