

For all questions, answer choice “E) NOTA” means none of the given answers is correct. Note that a normal distribution table is attached to the end of this test. Also, any question mentioning “die/dice” or “cards” or “coins” assumes that those objects are fair and standard, unless otherwise noted in the question.

- 1) A Macfarlane die is a special weighted die. It has sides numbered 1, 1, 2, 4, 4, and 6. With the exception of the side numbered 6, all other sides have an equal probability of occurrence of  $1/8$ . What is the expected value of rolling the Macfarlane die once? (Hint: the probability of a 6 occurring is  $3/8$ )

A) 3            B) 3.25            C) 3.5            D) 3.75            E) NOTA

- 2) Consider a special game using a standard die. In this game, a player will receive the face value (in dollars) of the side that is rolled. Hence, if a player rolls a 6, he or she will receive \$6. Thus, the expected value of this dice game is equal in dollars to the expected value of a standard dice roll. Consider an extension of this game. A player rolls once, and then is allowed to make a choice. He or she can either keep the money earned from the first dice roll, or he or she can decide to roll the dice again. If he or she decides to roll again, then the player will win the value of the side that is rolled the second time; the first roll's value is no longer relevant. What is the expected value (rounded to 2 decimal places) of this dice game where the player has the option to roll the dice twice?

A) 4.25            B) 4.33            C) 4.5            D) 4.67            E) NOTA

- 3) Don Pok Pa, a transfer student from Vietnam, is learning about probability distributions. Unfortunately, on his way home from school, it began to rain and the rain drops messed up his class notes. Below is all the information he could gather on two independent distributions,  $U$  and  $V$ .

$U$	0	1	2
$P(U = u)$	0.1	???	???

$V$	1	2	3
$P(V = v)$	???	0.2	???

In addition, Don Pok Pa recalls that  $P(U = 1, V = 1) = P(U = 1, V = 3) = 0.2$ . What is  $P(U = 2, V = 3)$ ?

- A) 0.04      B) 0.10      C) 0.16      D) 0.20      E) NOTA
- 4) Which of the following statements is FALSE regarding two non-zero probability events,  $G$  and  $H$ ?
- A)  $0 < P(G) \leq 1$       C)  $P(G|H) > 0$       E) NOTA  
 B)  $P(G \cup H) = P(G) + P(H) - P(G \cap H)$       D)  $P(G) + P(G^c) < 1$
- 5) Rosmerta, Ronald, Laguardia, Shakira, and Northmore are all vying for one of two coveted spots at the International Food Festival eating contest. If each pair of people is equally likely to go, what is the probability that neither Northmore nor Laguardia go?
- A)  $4/5$       B)  $7/10$       C)  $1/5$       D)  $3/10$       E) NOTA
- 6) Which of the following expressions represents the number of 2 card hands that can be made from a standard deck?
- A)  $\binom{2}{52}$       B)  $\frac{P_{52,2}}{2!}$       C)  $\frac{52!2!}{50!}$       D)  $\frac{P_{52,50}}{2!}$       E) NOTA

- 7) Which of the following statements is FALSE regarding combinations and permutations?
- A) Both combinations and permutations use sets of finite size.
  - B) Permutations account for order, whereas combinations do not.
  - C)  $P_{n,k} > \binom{n}{k}$ , where  $n > k > 1$  and  $n, k$  are positive integers.
  - D) Permutations allow for repeats, whereas combinations do not allow for repeats.
  - E) NOTA
- 8) Suppose  $\Omega$  is a continuous random variable uniformly distributed on the interval  $[0,4]$ . Compute the conditional probability that  $\Omega$  lies between 2 and 3 given that  $\Omega$  lies between 1 and 3, i.e. compute  $P(2 < X < 3 | 1 < X < 3)$ .
- A) 2/3      B) 5/8                      C) 1/2              D) 3/8                      E) NOTA
- 9) On the planet of Marklar, people, places, and things are all called Marklar. This system is quite confusing, so one day, a little Marklar decided to come up with a new naming system. All people, places, and things would now potentially be called either Marklar-1, Marklar-2, Marklar-3, or Marklar-4. Unfortunately, some Marklars fall in the intersection of these categories. Particularly, of the total 120 Marklar objects on Marklar, 12 are classified as Marklar-1 and Marklar-2, 34 are classified as Marklar-2 and Marklar-3, and 24 are classified as Marklar-1 and Marklar-4. There are no other overlaps. Also, a total of 50 objects can be called Marklar-1, and Marklar-3 contains 42 objects. Furthermore, if one were to pick a random Marklar out of a hat, the probability that the Marklar is only classified as Marklar-2 or Marklar-4 is  $\frac{1}{6}$ . What is the probability that a randomly selected Marklar does not fall into any of the Marklar-1,2,3, or 4 categories?
- A) 1/15                                      C) 1/30                                      E) NOTA  
B) 1/20                                      D) More Information Needed

- 10) Four rods A, B, C and D are to be welded end to end to make a 45 centimeter rod. The length of each of the rods is a normally distributed random variable with means  $\mu$  and standard deviations  $\sigma$  given in the table below. What is the probability that the assembled rods will be within 1 centimeter of 45 centimeters? Use the table provided on the last page and round to 4 decimal places.

Rod	A	B	C	D
$\mu$	10 cm	11 cm	12 cm	13 cm
$\Sigma$	1 cm	2 cm	2 cm	4 cm

- A) 0.0793      B) 0.1554      C) 0.3446      D) 0.6554      E) NOTA
- 11) Consider the equations  $2x + y = 1$  and  $2x + Ay = B$ , where  $A$  and  $B$  are the numbers obtained by rolling two fair dice, each numbered 1 through 6. What is the probability that these equations have at least one solution?
- A) 11/12      B) 31/36      C) 5/36      D) 1/12      E) NOTA
- 12) A player tosses two fair coins. The player wins \$3 if 2 heads occur and \$1 if 1 heads occurs. For the game to be fair, i.e. for the player's expected gain to be 0, how much should the player lose if no heads occurs? The player's LOSS should be:
- A) \$7      B) \$6      C) \$4      D) \$3      E) NOTA
- 13) Aunt Jemimah and Ron McDon are in an epic dice rolling battle! Here are the rules: Ron McDon goes first, and the first person to roll a 5 on a fair die wins. What is the probability that Ron McDon will lose this epic battle?
- A) 3/36      B) 1/4      C) 5/11      D) 6/11      E) NOTA

14) Mrs. Linder is a very sneaky lady and loves to give pop quizzes. On Monday, she has a 20% of giving one, assuming that the Tampa Bay Buccaneers won the Sunday before. She will normally give a pop quiz on Monday about 60% of the time. Interestingly, the Buccaneers have a 30% chance of winning if Mrs. Linder gives a pop quiz the week before. What is the probability that the Buccaneers win a random Sunday game (assume this value is constant and you have no information on when Mrs. Linder gave her last pop quiz?)

- A) 0.1      B) 0.4                      C) 0.7              D) 0.9                      E) NOTA

15) Consider the given information:  $P(A) = 0.3, P(B) = 0.5, P(A|B^c) = 0.2, P(B|A^c) = \frac{3}{7}$ . What is  $P((A \cap B^c) \cup (A^c \cap B))$ ?

- A) 0.4      B) 0.5                      C) 0.6              D) 0.7                      E) NOTA

16) Saleha possesses some very special coins. Her coin called Apple has a probability of  $\frac{1}{3}$  of producing a heads and her coin called Banana is a fair coin. Saleha flips each coin three times. Let  $X$  denote the number of heads produced by Apple and let  $Y$  denote the number of heads produced by Banana. Let  $Z = X + 2Y$ . What is the variance of  $Z$ ?

- A)  $\frac{145}{144}$       B)  $\frac{113}{72}$                       C)  $\frac{17}{12}$       D)  $\frac{11}{3}$                       E) NOTA

17) A point  $(x, y)$  is chosen at random in a rectangle 5 feet by 3 feet. What is the probability that the two coordinates  $x$  and  $y$  are within two feet of each other, i.e., compute  $P(|x - y| < 2)$ .

- A)  $\frac{11}{15}$                       B)  $\frac{2}{3}$                       C)  $\frac{8}{15}$                       D)  $\frac{1}{3}$                       E) NOTA

- 18) Two Geiger counters G and H are set to detect different radiation and are statistically independent. The counting rate is a Poisson process with an average counting rate of two clicks per minute for G and one click per minute for H. Given that G clicks less than three times in the first minute, what is the probability that G clicks less than H in the first minute (work out the cases where  $G = 0,1,2$ )? In other words, what is  $P(G < H | G \leq 2)$ ?

Hint: For a Poisson process (called X) with mean  $\mu$ ,  $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$ ,  $x = 0,1,2,3,\dots$

- A)  $1 - \frac{7}{2}e^{-3}$                       C)  $1 - 2e^{-2}$                       E) NOTA  
B)  $1 - e^{-2}$                       D)  $1 - 2e^{-1}$

- 19) Consider a square  $ABCD$ . A point  $E$  is picked at random along segment  $AB$  and a triangle is formed from points  $C, D, E$ . If another point  $F$  is picked at random from anywhere in  $ABCD$  or along the perimeter, what is the probability that  $F$  is not contained in triangle  $CDE$ ?

- A)  $1/8$       B)  $1/4$                       C)  $3/4$                       D)  $7/8$                       E) NOTA

- 20) Due to the uncertainty in expected sales for its new hover vehicle called Vector, WangGe Cars is considering conducting a marketing survey to determine customer attitudes toward the Vector and better predict the likelihood of strong sales (which WangGe predicts will occur 60% of the time). The marketing survey would give one of two results—a positive attitude or a negative attitude toward the design. WangGe has used this marketing survey for other vehicles and believes that each outcome is equally likely (half of vehicles receive a “Positive” result and half a “Negative result”). For those vehicles that receive a “Negative” result, they generate Strong sales only 40% of the time. If the Vector receives a “Positive” result from the survey, what would be the probability of Strong sales?

Hint: Use this alternate definition of probability.  $P(A) = \sum_{i=1}^n P(A|B_i)P(B_i)$ , where  $A$  is conditioned on a number of related events  $B$ .

- A) 50%      B) 70%                      C) 80%                      D) 90%                      E) NOTA

- 21) Jenn Hamburger, a fanatic gambler, plays a certain game. For each play, she will get \$10 if she wins and lose \$10 if she loses. Suppose that the probability to win is  $p$  ( $0 \leq p \leq 1$ ) and she begins playing with a given fortune \$100, what is the expected value after  $n$  independent plays of game?
- A)  $100 + 10n(2p - 1)$                       C)  $100 + 10n(1 - 2p)$                       E) NOTA  
B)  $100 + 10np(2p - 1)$                       D)  $100 + 10n(1 - p)$
- 22) Benihana and Jerimiah's K kosher Ice Cream is bankrupt! They used to have 33 flavors of ice cream, and now they can only afford 7. Fortunately, they still offer a special cup that allows the buyer to mix and match any of the 4 flavors. Even better, the buyer can repeat flavors as long as the total number of scoops is exactly four. How many different cups of ice cream can one customer possibly order?
- A) 210      B) 252                      C) 330                      D) 462                      E) NOTA
- 23) It has been determined that at the intersection of UPenn St. and Not Penn State Ave., cars arriving from the west go straight 10% of the time, turn left 70% of the time, and turn right 20% of the time. It is also known that 80% of drivers use their turn signals regularly (you can assume always) while 20% use them rarely (you can assume never). You, who are heading into the intersection from the west, are sitting behind a driver who does not have his turn signal on. What is the probability that he is turning left?
- A) 0.3      B) 0.5                      C) 0.6                      D) 0.7                      E) NOTA
- 24) Shmeagol, after losing his precious, decided to pick up golf as a pastime. He is only learning to play, and consequently his ball can fly in any number of directions. Assume that Shmeagol is at the origin of the Cartesian plane. His golf ball will fly on a straight line, and the range of slopes (all of which are equally likely to occur) for Shmeagol's trajectory is between  $y = \frac{1}{2}x$  and  $y = -\frac{1}{2}x$ . What is the approximate probability that he hits the ball along either  $y = 3x$  or  $y = -3x$ ?
- A)  $\frac{7}{8}$       B)  $\frac{3}{5}$                       C)  $\frac{1}{4}$                       D)  $\frac{1}{8}$                       E) NOTA

25) Students in a large class called Stochastic Processes take two midterms whose scores are approximately normally distributed. The first midterm has a mean of 75 and a standard deviation of 3. The second midterm has a mean of 85 and a standard deviation of 4. Find the approximate probability (using the attached Z-table) that the sum of the two scores for a student chosen at random will be larger than 165. Which of the following values is closest to that probability?

- A) 5%      B) 10%                      C) 15%      D) 25%                      E) NOTA

26) There are 2 red balls, 2 green balls and 1 yellow ball in a jar. Three balls are drawn out without replacement. What is the probability that there are at least as many red balls as there are yellow balls *left in the jar*?

- A)  $1/10$       B)  $2/5$                       C)  $4/5$       D)  $9/10$                       E) NOTA

27) Lazy John is an accountant preparing to take the CPA exam, which is a multiple choice test to evaluate a person's ability to reason. He uses a special computer program to study. The program randomly generates questions that Lazy John answers until he picks the incorrect choice for one of the questions. At this point, the computer will explain to him why he missed the question. Afterwards, he continues to answer another question in a similar manner as described before. Assuming that Lazy John has an equally likely chance of answering any randomly generated question when she starts, what probability distribution best describes this situation?

- A) Uniform                      C) Binomial                      E) NOTA  
B) Geometric                      D) Poisson

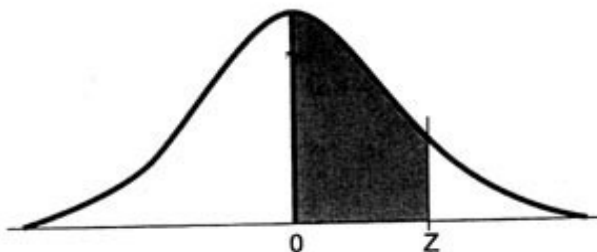
28) Which of the following is a discrete random variable?

- I. The average height of a randomly selected group of boys.
- II. The annual number of Lottery winners from New York City.
- III. The number of presidential elections in the 20th century.

- A) II only                      C) I and II only                      E) NOTA  
B) III only                      D) II and III only



- 29) Given that  $f(x) = x^2$ ,  $0 < x < 3$ , and each  $x$  value is equally likely to be chosen from this interval, what is  $P(f(x) < 2)$ ?
- A)  $1/2$       B)  $2/3$                       C)  $\sqrt{2}/3$       D)  $2/9$                       E) NOTA
- 30) If the odds that Palm Harbor University (PHU) wins the state competition sweepstakes are 5 to 3, what is the probability that PHU loses?
- A)  $3/8$       B)  $2/5$                       C)  $3/5$       D)  $5/8$                       E) NOTA



**Areas Under the Standard Normal Curve**

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999