1. Since each cycle of the Ferris wheel is 100π ft, the Ferris wheel has radius 50 ft. To be 75 ft above the ground for the first time, the Ferris wheel needs to rotate $120°$, or $\frac{1}{3}$ of a cycle, from its original position. Each cycle takes 6 minutes, so $\frac{1}{3}$ of a cycle takes 2 minutes, or $\boxed{120 \text{ seconds}}$.

2. It is not possible to number a cube with the integers from 1 to 6 without having at least one pair of adjacent sides with consecutive integers. So the probability sought is $\boxed{0}$.

3. Rearrange the product as $15^2 \cdot 25^2 \cdot 35^2 \cdot 45^2$. Compute the square of any two digit integer ending in 5 using

$$A5^2 = A(A+1) \cdot 100 + 25,$$

where A stands in for the tens digit. As seen in the formula, the tens digit and units digit of such a square will always be 2 and 5 respectively. Thus, the product will be of the form

$$w25 \cdot x25 \cdot y25 \cdot z25,$$

where the letters represent the number above the tens digit. Any such product will have a 2 and 5 in the tens and units digit respectively, so $\boxed{2+5=7}$.

4. The equation $(r, \theta) = (a \cdot sin(\theta), \theta)$ in the polar coordinates represents a circle of radius $\frac{a}{2}$ for the given range of θ. To get a circle of area 25, we need a radius of $\frac{a}{2} = \frac{5}{\sqrt{\pi}}$, and $\boxed{a = \frac{10}{\sqrt{\pi}}}$.

5. The key is recognizing that assigning negative integer values to x and y yields the greatest possible product. We can rearrange the given equation as

$$xy = 6 - 2x.$$

Clearly, minimizing x will maximize xy. Since x has to divide 6 for y to be an integer, the minimum x is -6. In that case, $xy = 6 - 2(-6) = \boxed{18}$.

6. If x, y, and z satisfy the given conditions, then $x = 4$, $y = 6$, and $z = 7$. Thus, $x + y + z = 4 + 6 + 7 = \boxed{17}$.

7. The angular frequency, ω, of a sum of trigonometric functions is the GCD of the constituent angular frequencies – in this case, 4π. Thus, the period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \boxed{\frac{1}{2}}.$$

8. Using the double-angle formula for sine,

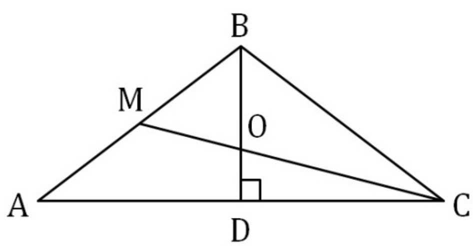
$$\sin\left(\frac{\pi}{16}\right)\cos\left(\frac{\pi}{16}\right)\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{4}\right)$$
$$= \frac{1}{2}\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{4}\right)$$
$$= \frac{1}{4}\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$$
$$= \frac{1}{8}\sin\left(\frac{\pi}{2}\right)$$
$$\boxed{= \frac{1}{8}}.$$

9. We can systematically make a list of all such numbers:

2311, 2317, 2319, 2371, 2379, 2971, and 2973

The sum is $\boxed{17641}$.

10. Point O is the centroid of triangle ABC, the point where the three medians intersect. Thus, O divides BD in a 2:1 ratio such that $\boxed{\frac{BO}{OD} = 2}$.



11. Let $AC = x$. By the triangle inequality, $1 \le x \le 7$. We can use the law of cosines to determine $\angle B$, where $\theta = \angle B$.

$$x^2 = 5^2 + 3^2 - 2(3)(5)\cos(\theta)$$
$$\cos(\theta) = \frac{5^2+3^2-x^2}{2(3)(5)} = \frac{34-x^2}{30}$$

If $\angle B$ is obtuse, $\cos(\theta) < 0$, so x must be 6 or 7. The sum: $6 + 7 = \boxed{13}$.

12. There are $\binom{25}{2} = 300$ pairs of distinct primes less than 100. If the product leaves a remainder of 1 when divided by 5, the units digit of the product must be either 1 or 6. We can divide the primes less than 100 into six different sets based on their units digits as follows.

$$S_1 = \{11, 31, 41, 61, 71\}$$
$$S_2 = \{2\}$$

$$S_3 = \{3, 13, 23, 43, 53, 73, 83\}$$
$$S_5 = \{5\}$$
$$S_7 = \{7, 17, 37, 47, 67, 97\}$$
$$S_9 = \{19, 29, 59, 79, 89\}$$

The product of any two elements of S_1, any two elements of S_9, or any element of S_3 with any element of S_7 results in a units digit of 1. This is a total of $\binom{5}{2} + \binom{5}{2} + 7 \cdot 6 = 10 + 10 + 42 = 62$ combinations. The product of any element in S_2 and any element of S_3 gives a units digit of 6. This adds $1 \cdot 7 = 7$ more combinations for a total of $62 + 7 = 69$ combinations. The probability is therefore $\boxed{\frac{69}{300} = \frac{23}{100}}$.

13. Integers a, b, c can be chosen in $6^3 = 216$ different ways. In order for vector $< b, c >$ with integer dimensions to be parallel to $< 1, a >$, it must be an integer multiple of the second. Thus,

$$< b, c > = k < 1, a > = < k, ak >$$

for some positive integer k. From the problem, $0 < ak \leq 6$, so

$$k = 1, 2, 3, 4, 5, or\ 6 \text{ when } a = 1,$$
$$k = 1, 2, or\ 3 \text{ when } a = 2,$$
$$k = 1\ or\ 2 \text{ when } a = 3,$$
$$k = 4 \text{ when } a = 4,$$
$$k = 5 \text{ when } a = 5, \text{ and}$$
$$k = 6 \text{ when } a = 6.$$

This is a total of 14 possibilities to yield a probability of $\boxed{\frac{14}{216} = \frac{7}{108}}$.

14. Let O be the origin. Since line m bisects the 30° angle made by l_1 and l_2, $\angle AOB = 15°$ and $\angle AOC = 75°$. Thus,

$$AB + AC = 5\sin(15°) + 5\sin(75°) = 5[\sin(15°) + \sin(75°)]$$
$$= 5\left[\sin\left(\frac{90-60}{2}\right) + \sin\left(\frac{90+60}{2}\right)\right]$$
$$= 5 \cdot 2\sin\left(\frac{90}{2}\right)\cos\left(\frac{60}{2}\right)$$
$$= 10\sin(45)\cos(30)$$
$$= 10 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \boxed{\frac{5\sqrt{6}}{2}}.$$

15. The roots of the equation are
$$\frac{2\cos(\theta) \pm \sqrt{4\cos^2(\theta) - \sin^2(2\theta)}}{2}$$

$= \cos(\theta) \pm \frac{\sqrt{4\cos^2(\theta) - 4\sin^2(\theta)\cos^2(\theta)}}{2}$

$= \cos(\theta) \pm \sqrt{\cos^2(\theta) - \sin^2(\theta)\cos^2(\theta)}$

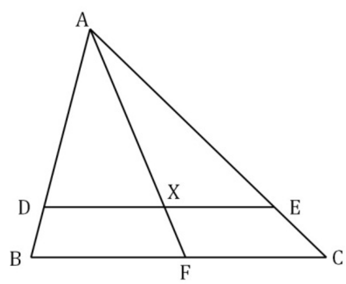
$= \cos(\theta) \pm \sqrt{\cos^2(\theta)(1 - \sin^2(\theta))}$

$= \cos(\theta) \pm \sqrt{\cos^4(\theta)}$

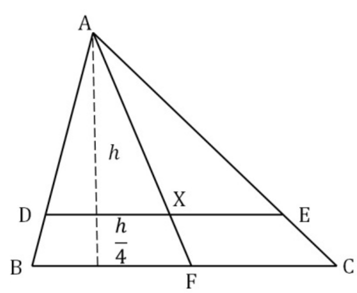
$= \cos(\theta) \pm \cos^2(\theta)$.

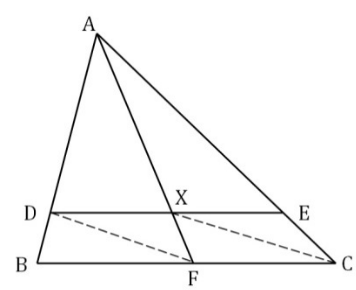
The greatest root is maximized for $\frac{\pi}{6} \le \theta \le \pi/2$ when $\cos(\theta)$ is maximized, or when $= \frac{\pi}{6}$. The maximum possible root is $\cos\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + \frac{3}{4} = \boxed{\frac{2\sqrt{3}+3}{4}}$.

16. First, notice the similar triangles in the diagram.



ΔADE is similar to ΔABC, with a ratio of 4:5, and ΔAXE is similar to ΔAFC, also with a ratio of 4:5. We can then draw the altitude from A to DX as having height h.





From properties of similar triangles, it follows that parallelogram $DFCX$ has height $\frac{h}{4}$. Since $DFCX$ is stated to be a parallelogram, it follows that $DX = FC$, and it also follows from similar triangles that $XE = \frac{4}{5}FC$. Then,

$$[ADE] = \frac{1}{2}h \cdot (DX + XE) = \frac{1}{2}h \cdot \frac{9}{5}DX,$$
$$[DFCX] = \frac{h}{4}DX, \text{ and}$$

$$\frac{[ADE]}{[DFCX]} = \frac{\frac{9}{10}h\cdot DX}{\frac{1}{4}h\cdot DX} = \boxed{\frac{18}{5}}.$$

17. Let (a_1, a_2), (b_1, b_2), (c_1, c_2), and (d_1, d_2) be the coordinates of A, B, C, and D respectively. Since A, B, C, and D all lie on a circle of radius 25 centered at the origin,

$$\sqrt{a_1^2 + a_2^2} = 25,$$
$$\sqrt{b_1^2 + b_2^2} = 25,$$
$$\sqrt{c_1^2 + c_2^2} = 25,$$
$$\sqrt{d_1^2 + d_2^2} = 25.$$

If A, B, C, and D are distinct points, then (a_1, a_2), (b_1, b_2), (c_1, c_2), and (d_1, d_2) simply represent the legs of distinct Pythagorean triangles with hypotenuse 25. There are exactly four such pairs that do not lie on the x or y-axis:

$$(7,24), (24,7), (15,20), \text{and } (20,15).$$

These points form a trapezoid with height $2\sqrt{2}$ and bases of length $5\sqrt{2}$ and $17\sqrt{2}$ for an area of $\boxed{2\sqrt{2}\cdot\frac{5\sqrt{2}+17\sqrt{2}}{2} = 44}$.

18. Let $<\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}}>$ and $<\frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}}>$ represent the unit direction vectors for the two lines. We can then make use of the property of the dot product:

$$<\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}}> \cdot <\frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}}> = 1\cdot 1\cdot \cos(\theta)$$

so $\cos(\theta) = \frac{8}{17}$ and $\sin(\theta) = \sqrt{1-\cos^2(\theta)} = \boxed{\frac{15}{17}}$.

19. Given $f(6) = f(7) = 2$, we can determine the next few terms.

$$f(8) = \left\lfloor \frac{7}{8}f(7)\right\rfloor + \left\lfloor \frac{6}{8}f(6)\right\rfloor = \left\lfloor \frac{14}{8}\right\rfloor + \left\lfloor \frac{12}{8}\right\rfloor = 2$$

$$f(9) = \left\lfloor \frac{8}{9}f(8)\right\rfloor + \left\lfloor \frac{7}{9}f(7)\right\rfloor = \left\lfloor \frac{16}{9}\right\rfloor + \left\lfloor \frac{14}{9}\right\rfloor = 2$$

We can actually determine that $f(k) = 2$ for all $k \geq 4$, so $\boxed{f(14) = 2}$.

20. Let $\theta = \sin^{-1}\left(\frac{1}{7}\right)$. Then, $\sin(\theta) = \frac{1}{7}$ and $\cos(\theta) = \sqrt{1-\sin^2(\theta)} = \frac{4\sqrt{3}}{7}$. Rearranging the given equation, we have

$$\sin^{-1}\left(\frac{1}{7}\right) + \cos^{-1}(x) = \pi$$
$$\cos^{-1}(x) = \pi - \sin^{-1}\left(\frac{1}{7}\right) = \pi - \theta$$

and finally

$$x = \cos(\pi - \theta) = -\cos(\theta) = \boxed{-\frac{4\sqrt{3}}{7}}.$$

21. Evaluating the expression, we have

$$\begin{aligned}&(\sin(\theta_1) + \cos(\theta_1))(\sin(\theta_2) + \cos(\theta_2))\\ &= \sin(\theta_1)\sin(\theta_2) + \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) + \cos(\theta_1)\cos(\theta_2)\\ &= \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2) + \cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)\\ &= \sin(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2) = \frac{24}{25} + \cos(\theta_1 - \theta_2)\end{aligned}$$

We can maximize the cosine term in the final expression by letting $\theta_1 = \theta_2$, so the maximum is $\frac{24}{25}$+$\cos(0) = \boxed{\frac{49}{25}}$.

22. We can narrow the range of θ with the first two parameters. For $-180^o \leq \theta \leq 180^o$,

I. $\sin(\theta) \leq \cos(\theta)$ implies $-135^o \leq \theta \leq 45^o$.

II. $\cot(\theta) \leq \tan(\theta)$ implies
$-135^o \leq \theta \leq -90^o$ or $-45^o \leq \theta \leq 0$ or $45^o \leq \theta \leq 90^o$ or $135^o \leq \theta \leq 180^o$.

The intersection of the first two ranges is $-135^o \leq \theta \leq -90^o$ and $-45^o \leq \theta \leq 0$. Upon inspection, we can also see that the range satisfies parameter III. Thus, the desired range is $\boxed{-135^o \leq \theta \leq -90^o}$.

23. The parameters of the problem define an ellipse as the area Mark's dog can travel. The ellipse has major axis of length 10 and minor axis of length 8, so the area is $\frac{10}{2} \cdot \frac{8}{2}\pi = \boxed{20\pi}$.

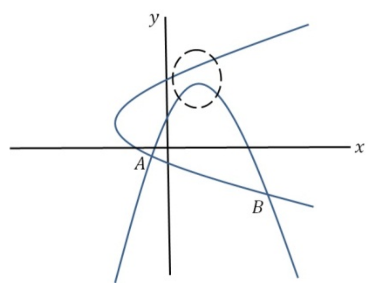
24. In this problem, we are essentially asked to find the 7th smallest nonnegative solution to the equation given. We can first consider the solutions to $|\sin(x) + 2\cos(x)| = 2$ for $0 \leq x < 2\pi$. Clearly, $x = 0$ and $x = \pi$ are solutions. When $\sin(x) \neq 0$, however,

$$\begin{gathered}|\sin(x) + 2\cos(x)| = 2\\ (\sin(x) + 2\cos(x))^2 = 4\\ \sin^2(x) + 4\sin(x)\cos(x) + 4\cos^2(x) = 4\\ \sin^2(x) + \cos^2(x) + 3\cos^2(x) + 4\sin(x)\cos(x) = 4\\ 1 + 3(1 - \sin^2(x)) + 4\sin(x)\cos(x) = 4\\ 1 + 3 - 3\sin^2(x) + 4\sin(x)\cos(x) = 4\end{gathered}$$

$$-3sin^2(x) + 4\sin(x)\cos(x) = 0$$
$$4\sin(x)\cos(x) = 3\sin^2(x)$$
$$\tan(x) = \frac{4}{3}.$$

$\tan(x) = \frac{4}{3}$ has two solutions for $0 \le x < 2\pi$, in the first and third quadrants of the coordinate plane, so there are four solutions to $|\sin(x) + 2\cos(x)| = 2$ for $0 \le x < 2\pi$. The 5th smallest nonnegative solution would be 2π, the 6th smallest would be $2\pi + \tan^{-1}\left(\frac{4}{3}\right)$ (in the first quadrant), and the 7th smallest would be $2\pi + \pi = 3\pi$. Thus, $\boxed{\alpha = 3\pi}$.

25. A rough sketch of y and y' superimposed should show that the two graphs intersect at least twice, below the x-axis. We arbitrarily label these points A and B as shown.



We now need to determine whether y and y' intersect above the x-axis, in the location enclosed by the dotted oval. We can do this analytically, by realizing that each point (x, y) in the graph of y corresponds to the point $(y, -x)$ in y'. We therefore make the appropriate substitution to arrive at the equation for y'.

$$(-x) = -(y)^2 + 2(y) + 2$$
$$-x = -y^2 + 2y + 2.$$

We can make the substitution $y = -x^2 + 2x + 2$ to find the x-values of the points of intersection of y and y'. So

$$-x = -y^2 + 2y + 2$$
$$x = y^2 - 2y - 2$$
$$x = (-x^2 + 2x + 2)^2 - 2(-x^2 + 2x + 2) - 2$$
$$0 = x^4 - 4x^3 + 8x + 4 + 2x^2 - 4x - 4 - 2 - x$$
$$0 = x^4 - 4x^3 + 2x^2 + 3x - 2$$
$$0 = (x - 1)(x^3 - 3x^2 - x + 2)$$

Since the resulting equation is quartic, there are at most four points of intersection, two of which we have already identified. Furthermore, we see that $x = 1$ is a solution to the quartic. $x = 1$ yields $y = 3$, so the point (1,3) is another point of intersection, this time above the x-axis. Does a 4th point of intersection exist? The fourth solution cannot be complex, because we have already identified three real solutions to the quartic, and there cannot be an odd number of complex solutions. The fourth solution is also not a repeated root. Thus, the number of points of intersection for y and y' is $\boxed{4}$.

26. We first divide the sum into two parts:

$$S = \sum_{i=0}^{x} i + \sum_{i=0}^{x}\left[\sin\left(\frac{\pi}{4}i\right) + \cos\left(\frac{\pi}{4}i\right)\right]$$
$$= \frac{x(x+1)}{2} + \sum_{i=0}^{x}\left[\sin\left(\frac{\pi}{4}i\right) + \cos\left(\frac{\pi}{4}i\right)\right]$$

The second summation is periodic over 8 terms, so its value oscillates as evidenced in the table below.

x	$\sum_{i=0}^{x}\left[\sin\left(\frac{\pi}{4}i\right) + \cos\left(\frac{\pi}{4}i\right)\right]$	x	$\sum_{i=0}^{x}\left[\sin\left(\frac{\pi}{4}i\right) + \cos\left(\frac{\pi}{4}i\right)\right]$
0	1	8	1
1	$1+\sqrt{2}$	9	$1+\sqrt{2}$
2	$2+\sqrt{2}$	10	$2+\sqrt{2}$
3	$2+\sqrt{2}$	11	$2+\sqrt{2}$
4	$1+\sqrt{2}$	12	$1+\sqrt{2}$
5	1	13	1
6	0	14	0
7	0	15	0

$S \geq 350$ for a specific value of x, when $\frac{x(x+1)}{2} + \sum_{i=0}^{x}\left[\sin\left(\frac{\pi}{4}i\right) + \cos\left(\frac{\pi}{4}i\right)\right] \geq 350$, which in turn implies that

$$\frac{x(x+1)}{2} + (2+\sqrt{2}) \geq 350 \text{ and thus}$$
$$\frac{x(x+1)}{2} \geq 348 - \sqrt{2} \sim 346.6$$

26 is the least value of x such that this expression is true. In fact, $\frac{26(27)}{2} = 351$, so the condition is satisfied regardless of the second sum, and so $\boxed{x = 26}$.

27. We have $f(x) = \frac{1}{(1-x)^2} = a_0 + a_1 x + a_2 x^2 + \cdots$ for $|x| < 1$. We manipulate this as follows:

$$\frac{1}{(1-x)^2} = a_0 + a_1 x + a_2 x^2 + \cdots$$
$$1 = (1 - 2x + x^2)(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots)$$
$$1 = a_0 + (a_1 - 2a_0)x + (a_2 - 2a_1 + a_0)x^2 + (a_3 - 2a_2 + a_1)x^3 + \cdots$$

In order for this equality to hold for all $|x| < 1$, a_0 must equal 1, and every coefficient in the right hand side must equal zero. Thus,

$$a_0 = 1,$$
$$a_1 - 2a_0 = 0,$$
$$a_2 - 2a_1 + a_0 = 0,$$
$$a_3 - 2a_2 + a_1 = 0, \text{ and}$$
$$a_4 - 2a_3 + a_2 = 0.$$

Solving gives $\boxed{a_4 = 5}$.

28. First, evaluate the given expression:

$$\tan[\arctan(u) + \arctan(v) + \arctan(w)]$$

$$= \frac{\tan(\arctan(u)+\arctan(v))+w}{1-\tan(\arctan(u)+\arctan(v))w}$$

$$= \frac{\frac{\mathrm{u+v}}{1-uv}+w}{1-\frac{\mathrm{u+v}}{1-uv}w} = \frac{u+v+w-uvw}{1-(uv+uw+vw)}.$$

From Vieta's formulas, we determine that

$$u + v + w = 4,$$
$$uv + uw + vw = -7, \text{ and}$$
$$uvw = -10,$$

so $\tan[\arctan(u) + \arctan(v) + \arctan(w)] = \frac{4+10}{1+7} = \boxed{\frac{7}{4}}$.

29. We can represent the problem in the following diagram:

$$o_T|a_1|a_2|a_3|o_B$$

The four vertical lines (|) represent the position of the four Aces in the deck. o_T and o_B represent the number of cards above and below the top and bottom-most Aces respectively, while a_1, a_2, and a_3 represent the number of cards between consecutive Aces. Finally, let

$$S_1 = a_1 + a_2 + 1,$$
$$S_2 = a_2 + a_3 + 1, \text{ and}$$
$$S_3 = a_1 + a_2 + a_3 + 2.$$

It follows that the set $\{x, 8, 12, 22, 26, 35\}$ corresponds in some manner to the set $\{a_1, a_2, a_3, S_1, S_2, S_3\}$. We next determine the correspondence.

Without loss of generality, we choose $a_3 \geq a_1$ (We will see that o_T and o_B do not play a role in the problem). Then,

$$S_3 > S_2 \geq S_1 > a_1,$$
$$S_3 > S_2 \geq S_1 > a_2, \text{ and}$$
$$S_3 > S_2 > a_3.$$

We now consider the correspondence by cases.

Case 1, $x > 35$:

We show this to be impossible. In this case, $S_3 = x$ and $S_2 = 35$. Since $S_1 > a_1, a_2$, S_1 cannot be one of the two smaller numbers, 8 or 12. However, S_1 does not necessarily equal 26, because a_3 can be larger than S_1. If $S_1 = 26$, we can see that no assignment of a_1 and a_2 from {8, 12, 22} satisfies the equation

$$S_1 = a_1 + a_2 + 1.$$

Similarly, no assignment of a_1 and a_2 from {8, 12, 22} satisfies the above equation for $S_1 = 22$. This case yields no solutions to the problem.

Case 2, $x \leq 35$:

It follows that $S_3 = 35 = a_1 + a_2 + a_3 + 2$, so

$$a_1 + a_2 + a_3 = 33.$$

The sum cannot be satisfied if one of the a-values equals 26, so the three values must be chosen from the set $\{x, 8, 12, 22\}$. This is only possible when $x = 3$ or $x = 13$. We can verify that both x-values satisfy the problem conditions by explicitly constructing the correspondences.

$$\{a_1 = 8, a_2 = 3, a_3 = 22, S_1 = 12, S_2 = 26, S_3 = 35\} \text{ and}$$
$$\{a_1 = 8, a_2 = 13, a_3 = 12, S_1 = 22, S_2 = 26, S_3 = 35\}$$

The answer is therefore $3 + 13 = \boxed{16}$.

30. We wish to find how many values of x exist for $0 \le x \le 4$ such that $f(x) = g(x)$. We can consider this more abstractly by considering the cases for θ_1 and θ_2 for $0 \le \theta_1, \theta_2$ such that $\sin(\theta_1) = \sin(\theta_2)$. There are three such cases:

Case 1:

$$\theta_1 = \theta_2$$

In this case, $\frac{\pi}{2}x = \frac{\pi}{2}x^2$, which is true for $x = 0, 1$. We can verify that $f(0) = g(0)$ and $f(1) = g(1)$, for **2** solutions.

Case 2:

$$\theta_2 = (2n-1)\pi - \theta_1 \quad \text{for } n = 1, 2, 3, \ldots$$

If $x < 1$, $\frac{\pi}{2}x^2 < \frac{\pi}{2}x$, so $f(x) = g(x)$ is satisfied when

$$\frac{\pi}{2}x = (2n-1)\pi - \frac{\pi}{2}x^2 \text{ or}$$
$$0 = x^2 + x + 4n - 2 \quad \text{for } n = 1, 2, 3, \ldots$$

However, there are no real solutions of x for $n = 1, 2, 3, \ldots$.

If $x \ge 1$, $\frac{\pi}{2}x \le \frac{\pi}{2}x^2$, and the reverse is true:

$$\frac{\pi}{2}x^2 = (2n-1)\pi - \frac{\pi}{2}x \text{ or}$$
$$0 = x^2 + x + 2 - 4n \quad \text{for } n = 1, 2, 3, \ldots$$

Thus real-valued solutions to

$$0 = x^2 + x - 2$$
$$0 = x^2 + x - 6$$
$$0 = x^2 + x - 10$$
$$0 = x^2 + x - 14$$
$$0 = x^2 + x - 18$$

are solutions to $f(x) = g(x)$. There are no real-valued solutions of x for $n > 5$ when $1 \le x \le 4$. Since each quadratic equation yields one real-valued solution, this case yields 5 solutions. However, the solution to the first quadratic is $x = 1$, which we have already considered in case 1, so this case yields **4** new solutions.

Case 3:

$$\theta_2 = \theta_1 + 2\pi n \text{ for } n = 1, 2, 3, \ldots$$

Again, there are no real solutions for when $x < 1$.
For $x \ge 1$, this case states that solutions to

$$\frac{\pi}{2}x^2 = \frac{\pi}{2}x + 2\pi n \text{ for } n = 1, 2, 3, \ldots \text{ or}$$
$$0 = x^2 - x - 4n \quad \text{for } n = 1, 2, 3, \ldots$$

are solutions to $f(x) = g(x)$. For $1 \le x \le 4$,

$$0 = x^2 - x - 4$$
$$0 = x^2 - x - 8$$
$$0 = x^2 - x - 12.$$

Each equation yields one unique real-valued solution, so this case gives **3** solutions.
In total, there are $2 + 4 + 3 = \boxed{9}$ solutions.

Tiebreaker Question:

1. Let n be a semiprime less than 100. Then neither prime factor of n can be greater than 50. The primes less than 50 are {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47}. The prime factorization of n can be one of

$$(2, 2 \leq p \leq 47) \text{ [15 combinations] or}$$
$$(3, 3 \leq p \leq 31) \text{ [10 combinations] or}$$
$$(5, 5 \leq p \leq 19) \text{ [6 combinations] or}$$
$$(7, 7 \leq p \leq 13) \text{ [3 combinations],}$$

where p is prime, for a total of $\boxed{15 + 10 + 6 + 3 = 34}$ possible semiprimes less than 100.