Practice Round Alpha State Bowl
Mu Alpha Theta National Convention 2013

P1. Find the common ratio of the geometric sequence $6, -42, \ldots$

P2. What is the area of the circle with equation $(x+1)^2 + (y-3)^2 - \frac{13}{\pi} = 0$?

P3. If the probability of event E happening is $\frac{5}{6}$, what are the odds against E happening? Express your answer as a common fraction.

P4. If θ is an acute angle such that $5 \sin\theta = 3$, find the value of $\sec\theta$ as a common fraction.

P5. Let A, B, C, and D be the answers to questions P1, P2, P3, and P4, respectively.
Evaluate: $\frac{AB}{CD}$

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Round #1 Alpha State Bowl
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1. Find x as a common fraction: $4+\sqrt{10-x}=6+\sqrt{4-x}$

2. Find the amplitude of the graph $y=2\cos x-2\sqrt{3}\sin x$.

3. If $\csc x=\frac{13}{\sqrt{7}}$, find $169\cos(2x)$.

4. Solve for x: $\log_2(2x)+\log_4 x+\log_8 x=12$

5. Let A, B, C, and D be the answers to problems 1, 2, 3, and 4, respectively.
Evaluate: $AB+C+\sqrt{D}$

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Round #2 Alpha State Bowl
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6. For integer n, let $\tau(n)$ equal the number of positive divisors of n. How many integers $N \in (0,200)$ satisfy the congruence $\tau(N) \equiv 1 \pmod 2$?

7. If x is a real number, find the number of solutions to $x + \sin x + e^x = 2$.

8. Evaluate: $2(\cos^2 0° + \cos^2 1° + \cos^2 2° + \cdots + \cos^2 89° + \cos^2 90°)$

9. What is the remainder when $2x^{603} - 3x^{250} + 10 - 6x^{25}$ is divided by $x + 1$?

10. Let A, B, C, and D be the answers to problems 6, 7, 8, and 9, respectively.
Evaluate: $\sqrt{A+2} + \sqrt{C + D - 2B}$

Round #2 Alpha State Bowl
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Round #3 Alpha State Bowl
Mu Alpha Theta National Convention 2013

11. Find, as a common fraction, the sum of all *real numbers* x such that $2x^3 + x^2 - 4 = 8x$.

12. Find the sum of the solutions to $\sin^2(5\theta) + \sin(2\theta) + \cos^2(5\theta) = 1$, where $\theta \in (\pi, 5\pi]$.

13. In triangle ABC, $m\angle C = \frac{\pi}{2}$ and $m\angle B = \theta$. If $\sec\theta = \frac{5}{3}$ and $|AB| = 15$, find the area of ABC.

14. What is the total surface area of a regular octahedron of volume 4/3?

15. Let A, B, C, and D be the answers to problems 11, 12, 13, and 14, respectively.
Evaluate: $A\tan^2 B + CD^2$

Round #3 Alpha State Bowl
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Round #4 Alpha State Bowl
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16. How many integers x satisfy $\big||x| - 7\big| \leq 8$?

17. The line with equation $2x - ky = 2013$ makes a $30°$ angle with the positive x-axis. Find k^4.

18. Let A, B, and C be the angle measures of a triangle. Let M be the maximum value of $\sin A \sin B \sin C$. Find the value of $128M^2$.

19. Find the product of all distinct complex numbers z with positive real part and $z^6 = -64$.

20. Let A, B, C, and D be the answers to problems 16, 17, 18, and 19, respectively.
Evaluate: $A + \sqrt{B} + \frac{2C}{D}$

Round #4 Alpha State Bowl
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Round #5 Alpha State Bowl
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21. Find the area of a quadrilateral with side lengths of 39, 52, 25, and 60 in that order.

22. A cube has volume of $\cos^3 x$ (where $0 < x < \frac{\pi}{2}$) and surface area of $36/17$. If $\sin^2 x = m/n$, where m and n are positive relatively prime integers, find $m + n$.

23. Find the number of degrees of the angle coterminal to $6912°$ in the interval $(0°, 360°)$.

24. If M and N are positive perfect cubes less than 1000 such that $M - N = 169$, find $M^{\frac{1}{3}} + N^{\frac{1}{3}}$.

25. Let A, B, C, and D be the answers to problems 21, 22, 23, and 24, respectively.
Evaluate: $A - B + C - D^2$

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Evaluate: $A - B + C - D^2$

Round #6 Alpha State Bowl
Mu Alpha Theta National Convention 2013

26. Let M be a 4×4 matrix such that $M \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} b \\ c/2 \\ 3d \\ a/4 \end{bmatrix}$ for all real numbers a, b, c, and d. Find the sum of the elements of $3M^{-1}$.

27. The domain of $f(x) = \sin^6 x + \cos^6 x$ is all real numbers x. The range of f is the interval $I = [a, b]$. Find the midpoint of I.

28. Find the number of petals in the polar graph $r = \sin(24\theta)$.

29. Find the distance from $(0,0)$ to the focus of the parabola with equation $8x + y^2 = 6y - 25$.

30. Let A, B, C, and D be the answers to problems 26, 27, 28, and 29, respectively.
Find the units digit of $\left(\frac{CB}{D}\right)^A$.

Round #6 Alpha State Bowl
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Round #7 Alpha State Bowl
Mu Alpha Theta National Convention 2013

31. Find $P(100)$, where $P(x)$ is a polynomial with real coefficients and $P(x^2) + 2x^2 + 10x = 2xP(x+1) + 3$ for all real x.

32. A triangle inscribed in the unit circle has angles measuring α, β, and γ. The perimeter of the triangle is 5. Evaluate: $\sin\alpha + \sin\beta + \sin\gamma$

33. Calculate $\text{Arcsin}(\sin 40° + \sin 20°)$ and express your answer in degrees.
Recall that $-90° \le \text{Arcsin}\, u \le 90°$ for $u \in [-1, 1]$.

34. The sequence $17, 20, 25, 32, \ldots$ has nth term given by $a_n = n^2 + 16$. Find the largest possible value of the greatest common divisor of two consecutive terms of this sequence as n ranges across the positive integers.

35. Let A, B, C, and D be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $A + BC + D$

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Round #8 Alpha State Bowl
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36. In triangle ABC with centroid P, let D and E be the foot of the medians to sides BC and AC, respectively. If AP is perpendicular to BE, $|AD| = 6$, and $|BE| = 9$, find the area of ABC.

37. Find the number of times the polar graph $r = 2^{\frac{2\theta}{\pi}}$ intersects the line segment whose endpoints are the Cartesian coordinates $(\sqrt{2}, \sqrt{2})$ and $(64\sqrt{2}, 64\sqrt{2})$.

38. Two sides of a triangle have length 8 and 15, while the sine of the acute angle between them is $\frac{8}{17}$. The measure of this angle is doubled while keeping the two side lengths the same, resulting in a new triangle. What is the ratio of the area of the *old triangle* to the new triangle? Express your answer as a common fraction.

39. Let a be a sequence such that $a_1 = 2$ and $a_n(1 - a_{n+1}) = 1$ for $n \geq 1$. Evaluate: $\sum_{n=1}^{833} a_n$

40. Let A, B, C, and D be the answers to problems 36, 37, 38, and 39, respectively.
Evaluate: $A + \frac{D}{30BC+2}$

Round #8 Alpha State Bowl
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Evaluate: $A + \frac{D}{30BC+2}$

Round #9 Alpha State Bowl
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41. Define $\Pi(S)$ as the product of the elements of a set S. Let $S_1, S_2, S_3, \dots, S_{15}$ be the nonempty subsets of $S = \{1, 2, 3, 4\}$. Evaluate: $\sum_{n=1}^{15} \left(\Pi(S_n)\right)^{-1}$

42. Find, in degrees, the measure of the smallest angle in a right triangle with legs of length a and b and hypotenuse of length $2\sqrt{ab}$, where a and b are positive numbers.

43. If $\sin u = \frac{3}{4}$, $\cos v = -\frac{1}{7}$, and $\tan w = 28$, evaluate: $12\sin(-u) - .5\cos(-v)\tan(-w)$

44. Let P be a point inside square $ABCD$ such that $|AP| = 5$, $|BP| = 2\sqrt{2}$, and $|CP| = 3$. Find the area of $ABCD$.

45. Let A, B, C, and D be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $10D - A^{B+C}$

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Round #10 Alpha State Bowl
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46. Find the sum of all positive integers n such that $\frac{2210}{(3n+5)(2n+3)}$ is an integer.

47. Find the smallest positive angle x (in radians) satisfying the equation
$$\left(\sin\left(\frac{2x}{3}\right)\cos\left(\frac{4x}{3}\right)+\cos\left(\frac{2x}{3}\right)\sin\left(\frac{4x}{3}\right)\right)\left(\cos\left(\frac{16x}{5}\right)\cos\left(\frac{6x}{5}\right)+\sin\left(\frac{16x}{5}\right)\sin\left(\frac{6x}{5}\right)\right)=\frac{1}{4}.$$

48. If $M=\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$, find the sum of the squares of the elements of M^{2013}.

49. Let $f(x)$ denote the integer closest to $\sqrt{x}$. Evaluate: $\sum_{n=1}^{650}\frac{1}{f(n)}$

50. Let A, B, C, and D be the answers to problems 46, 47, 48, and 49, respectively.
Evaluate: $AC+\frac{\pi}{B}+D$

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$$\left(\sin\left(\frac{2x}{3}\right)\cos\left(\frac{4x}{3}\right)+\cos\left(\frac{2x}{3}\right)\sin\left(\frac{4x}{3}\right)\right)\left(\cos\left(\frac{16x}{5}\right)\cos\left(\frac{6x}{5}\right)+\sin\left(\frac{16x}{5}\right)\sin\left(\frac{6x}{5}\right)\right)=\frac{1}{4}.$$

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