1. A

   n(n+1)/2 -> 197*198/2 = 19503

2. A

   BC = $\sqrt{AC^2 - AB^2} = 16$

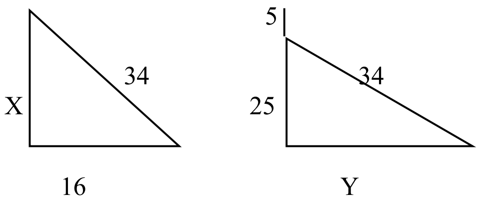
   12/BD = 20/(16-BD) ->

   192-12BD = 20BD

   192 = 32BD

   BD = 6, so that AD = $\sqrt{12^2 + 6^2} =$ $6\sqrt{5}$.

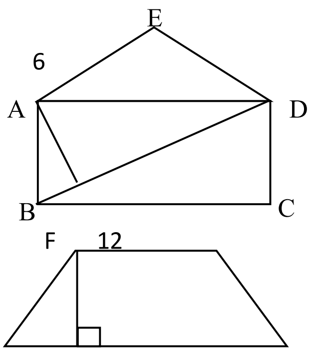
~~3. C~~



~~X = $\sqrt{34^2 - 16^2} = 30$~~

~~Y = $\sqrt{34^2 - 25^2} = 3\sqrt{59}$~~

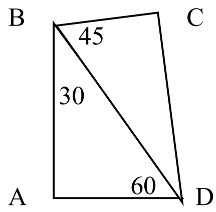
4. D



AF = $\sqrt{AF * FD} = 3\sqrt{5}$

AB = $\sqrt{AF^2 + BF^2} = 2\sqrt{6}$

5. A



m$\angle ABD = 180 - m\angle R - m\angle ADB = 30$

m$\angle CBD = 75 - 30 = 45$

DC = BC = $6\sqrt{6}$

AD = $6\sqrt{3}$ AB = 9

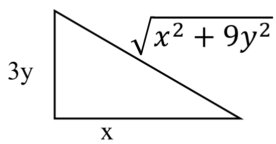
AD + AB + BC = 9+$6\sqrt{3}$+$6\sqrt{6}$

6. A

   S = 8 + $4\sqrt{2} + \frac{1}{2}(8 + 4\sqrt{2}) +$ $\frac{1}{4}(8 + 4\sqrt{2}) \ldots = \frac{8+4\sqrt{2}}{1-r} = \frac{8+4\sqrt{2}}{1-\frac{1}{2}}$

   $= \frac{8+4\sqrt{2}}{1-1/2} = 16 + 8\sqrt{2}$

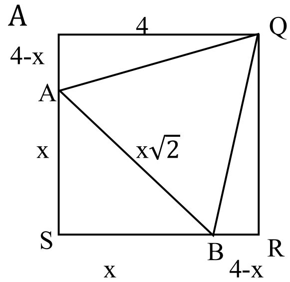
7. A



$\sqrt{x^2 + 9y^2}$

x/3y

8. A



$x\sqrt{2}$

QA = $\sqrt{4^2 + (4-x)^2}$
$= \sqrt{32 - x + x^2}$

AB = $\sqrt{2x^2}$

AB = QA

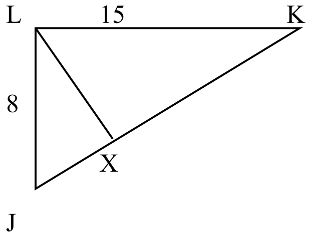
$\sqrt{2x^2} = \sqrt{32 - x + x^2}$

x = -4 + $4\sqrt{3}$

AB = $\sqrt{2}\,(-4 + 4\sqrt{3})$
$= 4\sqrt{2} + 4\sqrt{6}$

9. C -> Definition

10. E



$\sin\angle KLX + \csc\angle JLX$

$\frac{XK}{LK} + \frac{JK}{JX}$

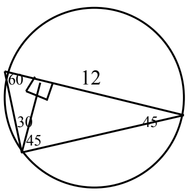
$\frac{KL*JL+JK^2}{JL*JK} = \frac{15*8+17^2}{8*17} = \frac{379}{136}$

11. E

Law of sins

$\frac{AB}{\sin C} = \frac{BC}{\sin A} = \sin\frac{75}{37}$

12. E



12 = x + x$\sqrt{3}$ -> x = $\frac{12}{1+\sqrt{3}}$

Perimeter = 12 +2x + x$\sqrt{6}$

= 12 + $\frac{24}{1+\sqrt{3}}$ + $\frac{12\sqrt{6}}{1+\sqrt{3}}$

= 6 + 3$\sqrt{6}$ + 9$\sqrt{2}$

13. B

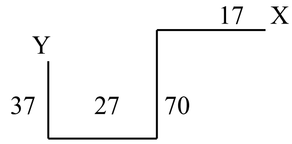
$\frac{12}{\sin\frac{\pi}{6}} = 2R$

R = 12

A/2 = 144 π/2 = 72 π

14. B – Definition

15. A



XY = $\sqrt{33^2 + 44^2}$ = 55

Mother's total time =

(33+27+70+17)*5 = 735 hours

V = $\frac{55}{734.5}$ = 110/1469

16. D

0.5*8*8$\sqrt{6}$ =32$\sqrt{6}$

17. A

$\sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-c/b}{2}} = \sqrt{\frac{b-c}{2b}}$

18. B

0 because the largest angle has to be across the largest side

19. B

Use the law of sins

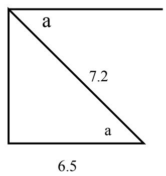
$\frac{b\sin C}{\sin b} = \frac{15\sin 57^\circ}{\sin 56^\circ}$

20. A

Heron's formula

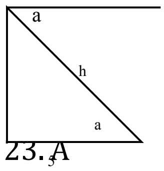
$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{176400}$

21. A



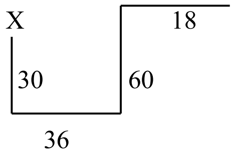
cos a = 6.5/7.2 = 0.9028

22. D



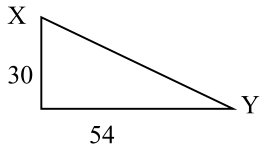
h = $\sqrt{12^2 + 5^2}$
= 13
sin a = 5/13
= .385

23. A



Bird A takes 6 hrs

Bird B takes 5.5 hrs



Direct distance = $\sqrt{30^2 + 54^2}$ =

= 6 $\sqrt{61}$

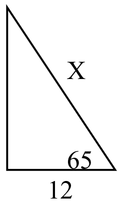
V = $6\frac{\sqrt{61}}{5.5} = 33\sqrt{61}$

24. D

A = 0.5absin a

= 0.5 * 10 * 12 sin (30) = 30

25. D

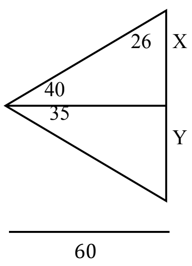


12/x = cos 65

X = $\frac{12}{\cos 65}$

$\frac{\frac{12}{\cos 65} ft}{5\ yd/min} * \frac{1\ yd}{3\ ft} * \frac{60\text{sec}}{1\text{ min}} = \frac{48\text{sec}}{\cos 65}$

26. C



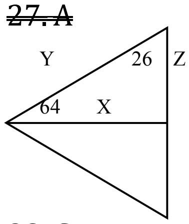
h = x + y

tan 40 = $\frac{x}{60}$

tan 35 = $\frac{y}{60}$

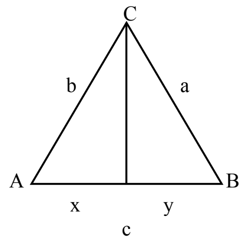
h = 60 tan 40 +
60 tan 35

~~27. A~~



Z = 2 * 60 = 120
Y = 3 * 60 = 180
X = $\sqrt{120^2 + 180^2} = 60\sqrt{13}$
$60\frac{\sqrt{13}}{60} = \sqrt{13} = 3.606$

28. C



x + y = c
bcosA + acosB = x + y
= 5100

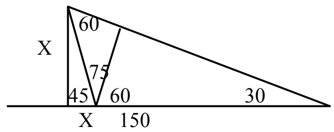
29. D

Cos BCX = cos CAB = 12/37

sec ACX = sec CBA = 37/35

$\frac{12}{37} + \frac{37}{35} = \frac{1789}{1295}$

30. D



$\sqrt{x^2 + (x+150)^2} = 2x$

$x^2 + (x+150)^2 = 4x^2$

$2x^2 - 300x + 22500 = 0$

By quadratic formula x = 289.78

Mu Alpha Theta National Convention: San Diego, 2013
Alpha Triangles Test - Updated Solutions

3 **(D)**. By the symmetry of isosceles triangles, all of I-IV are true. Item IV is true regardless, by the definition of a cevian.

9 **(C)**.

14 **(A)**.

18 **(B)**. All arguments are in degrees. By the Law of Sines, $20/\sin 30 = 16/\sin C$, or $\sin C = .40$. Since $\sin 30 = .50$, $C < 30$ or $C > 150$. It can't be the latter because we already have an angle that is equal to 30 degrees. Thus, there is only 1 possible triangle.

21 **(A)**. Since ABC is acute and scalene, all the points in the problem are distinct from each other. The nine-point circle only passes through p_1 through p_6, so the probability is $6/9 = 2/3$.

22 **(D)**. By Stewart's Theorem, $(10)(6)(10) + (10)(2)(10) = (x)(8)(x) + (2)(8)(6)$, or $x^2 = 88$.

23 **(B)**. By Ceva's Theorem, $(6/2)(10/5)(RT/TP) = 1$, or $RT/TP = 1/6$. Triangles QTR and QTP have the same base as measured from vertex Q. Thus, the ratio of their areas is the same as the ratio of their respective bases, or $1/6$. Thus, $m + n = 1 + 6 = 7$.

24 **(D)**. All arguments are in radians. Let $\alpha = m\angle A$ and $\beta = m\angle ABD = m\angle DBC = (\pi - \alpha)/4$. Using the Law of Sines, we have $BC = \frac{\sin\alpha}{\sin 2\beta}$, $BD = \frac{\sin\alpha}{\sin 3\beta}$, and $AD = \frac{\sin\beta}{\sin 3\beta}$. Since $AD + DB = BC$, this yields the equation $\sin(\pi - 4\beta)\sin(3\beta) = (\sin(\pi - 4\beta) + \sin\beta)\sin(2\beta)$, which turns into $\sin(2\beta)\sin(5\beta) = \sin(2\beta)\sin(4\beta)$ after application of the sum-to-product identities. Thus, $\sin(5\beta) = \sin(4\beta)$, making $\beta = \pi/9$ and $\alpha = 5\pi/9$, or 100 degrees.

25 **(B)**. All angles are in degrees. We have $m\angle C = 75$. The inscribed triangle that will yield the least perimeter is the orthic triangle, and its area is given by $K_{\text{orthic}} = \frac{abc|\cos A\cos B\cos C|}{2R}$, where R is the circumradius of triangle ABC. Compare this with the formula for the area of ABC, $K = \frac{abc}{4R}$, and we see that the desired ratio is equal to $2\,|\cos A\cos B\cos C|$. The answer is $2\cos 45\cos 60\cos 75 = (\sqrt{3} - 1)/4$.

26 **(C)**. In a triangle, $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$. Thus, the function in the problem is identically 1, which is both the minimum and the maximum.

27 **(B)**. A straightforward calculation shows that a disc of radius 3 and 4 can be fit into two corners of an equilateral triangle with side $11\sqrt{3}$ as to just touch. The disc of radius 2 will easily fit into the third corner without overlapping with the other two discs. Thus, $L = 11\sqrt{3}$ and $L^2 = 121(3) = 363$, making for a digital sum of $3 + 6 + 3 = 12$.

29 **(D)**. All angles are in degrees. Interpret each equation as applying the Law of Cosines to a particular set of triangles that are "stuck" together via a common vertex. In particular, consider triangle ABC and a point P inside the triangle such that $AP = x$, $BP = y$, $CP = z$, $m\angle APB = 90$, $m\angle APC = 120$, and $m\angle CPB = 150$. The desired expression is simply 4 times the area of ABC in terms of x, y, and z. In this case, ABC happens to be a right triangle with legs of $\sqrt{7}$ and $\sqrt{21}$, so its area is $(1/2)(\sqrt{7})(\sqrt{21}) = 7\sqrt{3}/2$. The answer is $14\sqrt{3}$.