



Individual Relay

Test #711 (Mu), #721 (Alpha), #731 (Theta)

1. Write your 6-digit ID# in the ID# blank and circle your division. In the Name blank, print your name; in the School blank, print your school name (no abbreviations).
2. This test consists of five Relays, each consisting of six questions. Each question in a Relay, except for the first one, references the answer from previous questions in the Relay in some form. All answers to the questions on this test are integers.
3. Scoring is as follows: The six questions in each Relay are worth, in order, 2, 3, 4, 5, 6, and 7 points. There is no penalty for incorrect answers; however, total points for a Relay stop being earned at the first incorrect or blank answer. For example, a test-taker who has answered the first two questions correct in a Relay, but gets the third question wrong or leaves it blank will receive a total of $2+3=5$ points for that Relay. This is true even if the test-taker happens to answer the fourth or fifth questions correctly. Additionally, a Relay whose all six questions are answered correctly will be granted an additional 3 points as a bonus. The maximum attainable score in one Relay is 30 points.
4. You may not sit adjacent to anyone from your school.
5. **TURN OFF ALL CELL PHONES OR OTHER PORTABLE ELECTRONIC DEVICES NOW.**
6. No calculators may be used on this test.
7. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future national conventions, disqualification of the student and/or school from this convention, at the discretion of the Mu Alpha Theta Governing Council.
8. If a student believes a test item is defective, file a Dispute Form explaining why.
9. Unless a question asks for an approximation or a rounded answer, give the exact answer.

Name: _____

ID#: _____ School: _____

Division / Test ID# (Circle One): Mu (#711) Alpha (#721) Theta (#731)

Relay 1:

Q#	Answer	Points	Q#	Answer	Points	Bonus?	R1 Total
1)		2 0	4)		5 0	3 0	
2)		3 0	5)		6 0		
3)		4 0	6)		7 0		

Relay 2:

Q#	Answer	Points	Q#	Answer	Points	Bonus?	R2 Total
1)		2 0	4)		5 0	3 0	
2)		3 0	5)		6 0		
3)		4 0	6)		7 0		

Relay 3:

Q#	Answer	Points	Q#	Answer	Points	Bonus?	R3 Total
1)		2 0	4)		5 0	3 0	
2)		3 0	5)		6 0		
3)		4 0	6)		7 0		

Relay 4:

Q#	Answer	Points	Q#	Answer	Points	Bonus?	R4 Total
1)		2 0	4)		5 0	3 0	
2)		3 0	5)		6 0		
3)		4 0	6)		7 0		

Relay 5:

Q#	Answer	Points	Q#	Answer	Points	Bonus?	R5 Total
1)		2 0	4)		5 0	3 0	
2)		3 0	5)		6 0		
3)		4 0	6)		7 0		

Grand Total: _____

This test consists of five relays of six questions each. Denote the answer to question number q in Relay r as $N(r, q)$. So, for example, $N(3,1)$ is the answer to the first question in Relay 3.

All answers on this test are integers.

Relay 1:

1. If a and b are two positive integers that satisfy $a^2 - b^2 = 2013$, let M and N be the largest and smallest possible values for $a + b$, respectively. What is $2000 - M + N$?
2. Let $T = N(1, 1)$. Define a function $S(x)$ as the sum of all positive integral factors of x . Find $S(T)$.
3. Let $T = N(1, 1)$ and $U = N(1, 2)$. If $f(x)$ is a quadratic function with $f(1) = T$, $f(2) = 79$, and $f(3) = U$, what is the y -intercept of the graph of $y = f(x)$?
4. Let $T = N(1, 3)$. How many positive prime numbers p less than T are there such that $p + 50$ is also a prime number?
5. Let $T = N(1, 4)$. A spider at $(1, 1)$ can only move up or right one unit each time, but not left or down. How many ways are there for the spider to reach (T, T) without passing through $(2, 2)$?
6. Let $T = N(1, 5)$. My favorite number is between 100 and 300. When my favorite number is divided by T , the remainder is $0.6T$; divided by 7, the remainder is 6. What's my favorite number?

Relay 2:

1. What's the distance from the point $(1, 1)$ to the line $5x - 12y = 32$?
2. Let $T = N(2, 1)$. Parabolas $f_1(x) = x^2 - Tx - 2T$ and $f_2(x) = -x^2 + 9x - 20$ have two intersections, (x_1, y_1) and (x_2, y_2) . Find $x_1 + x_2$.
3. Let $W = N(2, 1)$ and $U = N(2, 2)$. If the addition below is done correctly and different letters represent distinct digits, find the sum of the digits *not* used.

$$\begin{array}{r} T W O \\ + T W O \\ \hline F O U R \end{array}$$

4. Let $T = N(2, 3)$. The prime factorization of $T!$ is $2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdot \dots$. Calculate $a/c + b/d$.
5. Let $T = N(2, 4)$. A non-negative integer n is called b -friendly if, when n is written in base b , it only contains 0s and 1s. What is the T -th smallest T -friendly number in base 10? (0 and 1 are always friendly numbers)
6. Let $T = N(2, 5)$. Let $S = \lfloor \sqrt{T} \rfloor$, where $\lfloor x \rfloor$ is the floor function, the largest integer less than or equal to x . If $i = \sqrt{-1}$, a and b are real numbers, and

$$\sum_{n=1}^S (1+i)^n = a + bi$$

Find $a + b$.

Relay 3:

1. Triangle ABC has $\angle C = 90^\circ$. There is a point D on \overline{AC} such that $4 \cdot \angle CBD = \angle DBA$. If $\overline{AC} \cdot \overline{DC} = \overline{BC}^2$, find $\angle CAB$ in degrees.
2. Let $T = N(3, 1)$. What's the smaller eigenvalue of $A = \begin{bmatrix} T & T+3 \\ T+3 & 2T \end{bmatrix}$?
(An eigenvalue λ is a scalar value such that $Ax = \lambda x$ for some 2×1 matrix x .
 $\det(A - \lambda I) = 0$, where I is the identity matrix.)
3. Let $T = N(3, 2)$. The side length of a regular T -gon is $2T$. Find the square of its area.
4. Let $T = N(3, 1)$ and $S = N(3, 3)$. There are S coins on a table. Connie and David are playing a game: each person has to take at least 1 coins but at most T coins every turn, and whoever collects the last one wins. If David is first to act, how many coins should he take to guarantee a victory?
5. Let $T = N(3, 1)$ and $S = N(3, 4)$. If a, T, c form an arithmetic sequence, b, S, d form a geometric sequence, b, S, c form an arithmetic sequence, a, T, d form a geometric sequence, compute $a + d + bc$.
6. Let $T = N(3, 5)$. Find the units digit of $2013^T + T^{2013}$.

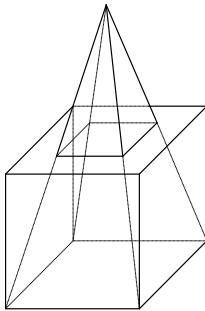
Relay 4:

1. A train is traveling at a constant speed 40mph from Town A to Town B . If the speed is changed to 50mph halfway between the towns, then its traveling time will be shortened by an hour. Find the distance between A and B in miles.

- Let $T = N(4, 1)$. Catherine bought T toys and \sqrt{T} candies for her children. She tried to give them an equal number of toys and candies, but found herself in need of 7 more toys and 2 more candies. How many children does she have?
- Let $T = N(4, 2)$. A $3 \times 3 \times 3$ Rubik's cube consisting of 27 unit cubes has 18 unit cubes with 1 or 2 visible surfaces: 6 centers and 12 edges. How many unit cubes in a $T \times T \times T$ Rubik's cube would have 1 or 2 visible surfaces?
- Let $T = N(4, 3)$. What is the minimum positive value u such that $(T + u)/100$ is an integer?
- Let $T = N(4, 4)$. Find the area of the triangle bounded by the x -axis, $x + Ty = T$, and $3x + 2y = -T$.
- Let $T = N(4, 5)$. From a standard deck of 52 cards, Allen randomly draws T cards. The ratio of the probability that he sees exactly 1 spade in his hand to the probability that he sees no spades is $m:n$, where m and n are positive and relatively prime. Find $m + n$.

Relay 5:

- A circle C with radius $r = \sqrt{425}$ is centered at the origin and goes through n lattice points. Find $n/2$.
- Let $T = N(5, 1)$. In the diagram below, a square pyramid with height $2T$ is placed inside a cubic box with side length T . What is the volume inside the box but outside the pyramid?



- Let $T = N(5, 2)$. What is the real solution of $x = \sqrt[3]{T + \sqrt[3]{T + \sqrt[3]{T + \dots}}}$?
- Let $T = N(5, 3)$. What is the reciprocal of the probability that when two fair six-sided dice are rolled the values on the top faces add up to over T ?
- Let $A = N(5, 1)$, $B = N(5, 2)$, $C = N(5, 3)$, $D = N(5, 4)$. The greatest common divisor of A and B is g ; the least common multiple of C and D is l . Compute $\sqrt{|g - l| + 3}$.

6. Let $A = N(5, 1)$, $B = N(5, 2)$, $C = N(5, 3)$, $D = N(5, 4)$, $E = N(5, 5)$. For $x, y \in \mathbb{R}$,
 $f(x, y) = Ex^2 + Dxy + Cy^2 + Ax + B$. Find the minimum value of this function $f(x, y)$.