

1. $x = 1.01, y = 0 + 10 \cdot (1 + 0) \cdot .01 = .1; x = 1.02, y = .1 + 10 \cdot (1.01 + .1) \cdot .01 = .211; x = 1.03, y = .211 + 10 \cdot (1.02 + .211) \cdot .01 = \boxed{.3341}$
2. Width x , length y , $2x + y = 60, y = 60 - 2x$, area = $xy = x(60 - 2x), \frac{dA}{dt} = -4x + 60 = 0, x = 15, y = 30, A = \boxed{450}$
3. $\frac{dP}{dt} = 1200e^{0.75t}, P = 1600e^{0.75t} + C, P(0) = 4800 \Rightarrow C = 3200, P(t) = 1600e^{0.75t} + 3200$. When $P(t) = 9600, 1600e^{0.75t} = 6400, e^{0.75t} = 4, t = \boxed{\frac{4}{3} \ln 4}$
4. $\frac{dP}{dt} = 0.001P^2, P(t) = \frac{1}{c - 0.001t}, P(0) = 100, C = 0.01, P(5) = \frac{1}{0.01 - 0.005} = \frac{1}{0.005} = \boxed{200}$
5. Center of mass = $\frac{\int_0^{20} r \rho dr}{\int_0^{20} \rho dr} = \frac{\int_0^{20} r^3 / 400 dr}{\int_0^{20} r^2 / 400 dr} = \frac{\int_0^{20} r^3 dr}{\int_0^{20} r^2 dr} = \frac{\frac{r^4}{4} \Big|_0^{20}}{\frac{r^3}{3} \Big|_0^{20}} = \frac{\frac{20^4}{4}}{\frac{20^3}{3}} = \boxed{15}$
6. $I = \int_0^{20} r^2 \rho dr = \frac{1}{400} \int_0^{20} r^4 dr = \frac{1}{2000} r^5 \Big|_0^{20} = \boxed{1600}$
7. Arclength = $\int_0^{\pi/4} \sqrt{1 + \tan^2 x} dx = \int_0^{\pi/4} \sec x dx = \ln|\tan x + \sec x| \Big|_0^{\pi/4} = \ln \left| \tan \frac{\pi}{4} + \sec \frac{\pi}{4} \right| - \ln|\tan 0 + \sec 0| = \boxed{\ln|1 + \sqrt{2}|}$
8. Vol = $\int_0^4 \left(\sqrt{\frac{4-y}{2}} \right)^2 \pi dy = \pi \left(2y - \frac{1}{4}y^2 \right) \Big|_0^4 = \boxed{4\pi}$
9. $v(t)$ has to be continuous, $v(t) = \begin{cases} \frac{1}{2}t^2, & t < 3 \\ 3t - 4.5, & t \geq 3 \end{cases} \Rightarrow \frac{1}{5} \int_0^5 v(t) dt = \boxed{3.9}$
10. $e^{-\frac{\ln 2}{200}t} = 0.01 = e^{-\ln 100}, t = \frac{400 \ln 10}{\ln 2} \approx \frac{920}{0.7} \approx 1314 \Rightarrow \boxed{3314}$
11. Set $\overline{AD} = 5 - x$. Cost = $30(5 - x) + 50\sqrt{1 + x^2}, -30 + \frac{50x}{\sqrt{1+x^2}} = 0, 5x = 3\sqrt{1 + x^2}, 16x^2 = 9, x = 0.75, \overline{AD} = 5 - x = \boxed{17/4}$
12. profit = $P(w) \cdot w - C(w) = -2w^2 + 60w - 100, -4w + 60 = 0, w = \boxed{15}$
 In economics terms, revenue = $P(w) \cdot w$, marginal revenue (MR , the additional revenue from selling one more unit) = $\frac{dR}{dw} = 60 - 2w$; marginal cost (MC , the additional cost from making one more unit) = $\frac{dC}{dw} = 2w$; optimal choice is when $MR = MC, w = 15$

13. profit = $P(2w) \cdot w - C(w) = -3w^2 + 60w - 100$, derivative = $-6w + 60 = 0$, $w =$

$\boxed{10}$, market output = $2 \cdot 10 = \boxed{20}$

$MR = 60 - 4w = MC = 2w$, so $w = 10$

14. $= \int_1^4 x e^{3x} dx = \left(\frac{1}{3}x - \frac{1}{9}\right) e^{3x} \Big|_1^4 = \boxed{\frac{11}{9}e^{12} - \frac{2}{9}e^3}$

15. $a = \frac{d^2x}{dt^2} = -4x$ (negative because of direction), $x = m \cos 2t + n \sin 2t$, initial

conditions yield $m = 5, n = 0, t = 4 \Rightarrow x(4) = \boxed{5 \cos 8}$

16. $W =$ hours at work, $C = 8W, L = 24 - W, U = CL = 192W - 8W^2, \frac{dU}{dW} = 192 -$

$16W = 0, W = \boxed{12}$

17. Consider the cross section, and suppose the trapezoid touches the tunnel at

$(\pm x, 9 - x^2), (\pm 3, 0)$. Volume of prism = $\frac{1}{2}(2x + 6)(9 - x^2)(6) = 3(-2x^3 - 6x^2 +$

$18x + 54), \frac{dV}{dx} = 3(-6x^2 - 12x + 18) = -18(x + 3)(x - 1) = 0, x = 1, V =$

$\frac{1}{2}(8)(8)(6) = \boxed{192}$

18. $C = 0.01s^2 \cdot 5s^3 + \frac{3600}{s}, \frac{dC}{ds} = 0.25s^4 - \frac{3600}{s^2} = 0, s^6 = 14400, s = \sqrt[3]{120},$ speed =

$\boxed{10\sqrt[3]{120}}$

19. $= \frac{1}{10} \int_3^6 0.01s^2 \cdot 5s^2 dx = \frac{1}{1000} s^5 \Big|_3^6 = \boxed{7.533}$

20. $U = 2\sqrt{F} + C = 2\sqrt{F} + \frac{60-4F}{6}, \frac{dU}{dF} = \frac{1}{\sqrt{F}} - \frac{2}{3} = 0, F = \left(\frac{3}{2}\right)^2 = \frac{9}{4}, C = \frac{60-4 \cdot \frac{9}{4}}{6} = \boxed{8.5}$

In economics, $\frac{dU/dF}{dU/dC} = \frac{\text{price of food}}{\text{price of clothing}}, \frac{1/\sqrt{F}}{1} = \frac{2}{3}, F = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

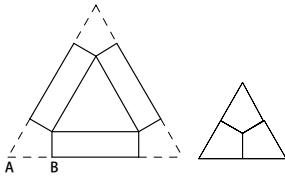
21. The volume of a regular octahedron is $\frac{1}{3}\sqrt{2}s^3$, where s is the side length. When $V =$

$9\sqrt{2}, s = 3, \frac{d}{dt}(SA) = \frac{d}{dt}\left(8 \cdot \frac{\sqrt{3}}{4}s^2\right) = 2\sqrt{3} \cdot 2s \cdot \frac{ds}{dt} = 4\sqrt{3} \cdot 3 \cdot 0.25 = \boxed{3\sqrt{3}}$

22. Total probability = $\int_0^1 k(-2x^3 + 3x^2 + 6x + 1)dx = \frac{9k}{2} = 1, k = \frac{2}{9}$

Expected value = $\int_0^1 \frac{2}{9}(-2x^3 + 3x^2 + 6x + 1)xdx = \boxed{\frac{19}{30}}$

23. Set $\overline{AB} = x$, Volume = $\frac{x}{\sqrt{3}} \cdot \frac{\sqrt{3}}{4} (18 - 2x)^2 = x^3 - 18x^2 + 81x$, $\frac{dV}{dx} = 3x^2 - 36x + 81 = 3(x - 3)(x - 9)$, $x = 3, 9$. Volume is 0 when $x = 9$, a local min. $x = 3$ is a local max, and the three removed regions can combine to form an equilateral triangle whose side length is 6, which would be $\boxed{\frac{1}{9}}$ of the original area.



24. $\frac{dP}{dt} = r(M - P) - kP = -P + 800$, $P(t) = Ce^{-t} + 800$, $P(0) = 300 \Rightarrow C = -500$,
 $P(3) = \boxed{-500e^{-3} + 800}$.

25. $V = bh$, $\frac{dV}{dt} = b \frac{dh}{dt} = 2\sqrt{20} \frac{dh}{dt} = -2v = -2\sqrt{2gh}$, $\frac{dh}{dt} = -\sqrt{h}$, $h = (C - 0.5t)^2$, $h(0) = \frac{200\sqrt{20}}{2\sqrt{20}} = 100$, $C = 10$, $h = 0$ when $t = \boxed{20}$

26. Let $x(t)$ and $y(t)$ be the amount of ethanol in tank 1 and tank 2, respectively. Then,

$$\frac{dx}{dt} = -\frac{10}{100}x, \frac{dy}{dt} = \frac{10}{100}x - \frac{10}{100}y; x = Ce^{-0.1t} \xrightarrow{x(0)=100} x = 100e^{-0.1t}; \frac{dy}{dt} + 0.1y = 10e^{-0.1t}, \frac{d}{dt}(ye^{0.1t}) = e^{0.1t} \left(\frac{dy}{dt} + 0.1y \right) = 10, ye^{0.1t} = 10t + C \xrightarrow{y(0)=0} y = 10te^{-0.1t}, y(5) = \boxed{50e^{-0.5}}$$

27. Let d be the biking distance. Time + cost = $P = \frac{d}{4} + \frac{30-d}{40} + \left(\frac{d}{4}\right)^2 + 5 + \frac{(30-d)^2}{5}$, $\frac{dP}{dd} = \frac{1}{4} + \frac{-1}{40} + \frac{d}{8} + \frac{-2(30-d)}{5} = \frac{-471}{40} + \frac{21d}{40} = 0$, $d = \boxed{\frac{157}{7}}$

28. $r = e^{\frac{3}{4}\theta}$, arclength = $\int_0^4 \sqrt{e^{\frac{3}{2}\theta} + \frac{9}{16}e^{\frac{3}{2}\theta}} d\theta = \int_0^4 \frac{5}{4}e^{\frac{3}{4}\theta} d\theta = \frac{5}{3}e^{\frac{3}{4}\theta} \Big|_0^4 = \boxed{\frac{5}{3}(e^3 - 1)}$

29. $\int_0^{a_0} P(r)dr = -\frac{e^{-2r/a_0} \cdot (a_0^2 + 2a_0r + 2r^2)}{a_0^2} \Big|_0^{a_0} = \boxed{1 - 5e^{-2}}$

30. $\frac{dy}{dx} = \frac{9(1-x^2)}{3} = 3(1-x^2)$, $y = 3x - x^3 + C$, $y(-1) = 0 \Rightarrow C = 2$, $y(1) = 3 - 1 + 2 = \boxed{4}$