Mu Individual Solutions:

1. Since f(x) is continuous and differentiable on [1,4], then the slope at some point on the interval's interior has a slope equal to the slope connecting (1,f(1)) and (4,f(4)), or (1,1) and (4,2). This slope is 1/3.

2. The derivative of ln(x) is 1/x. Therefore the derivative of 1/ln3 * ln(x) = 1/(x*ln3). Evaluated at x = 2, this is 1/(2ln3) = 1/ln9.

3. This is an application of the quotient rule. When h(x) = f(x)/g(x), h'(x) = (f'(x)g(x)-g'(x)f(x))/g^2(x).

4. Let $u = 2t - 3$, then $du = 2dt$. $\int \frac{1}{2}u^4 du = \frac{1}{10}u^5 + C$. Substituting back in, this equals $\frac{1}{10}(2t-3)^5 + C$.

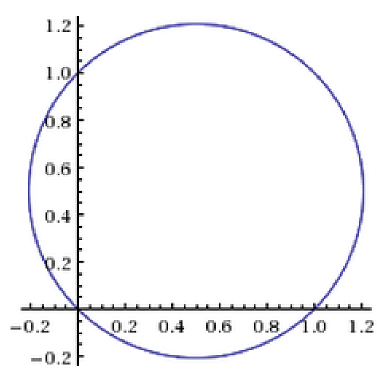
5. You first must calculate f(1), f'(1) and f''(1). f(1) = 1; f'(1) = ½; f''(1) = -1/4. So $p_2(x) = 1 + .5(x-1) - .125(x-1)^2$. Evaluating at x = 1.3: 1+0.15-.01125 = 1.13875 -> rounds to 1.139.

6. This is exactly Rolle's Theorem.

7. f can only have a relative minimum when f' = 0, which is only when x = 0,-2,3. However, there is a maximum at -2 since f''(-2) < 0, and 0 is neither a max nor a min.

8. The numerators decrease by 3 and the denominators increase by 4, so the infinite limit of the ratios is -3/4.

9. In polar, use the formula $A = \int_a^b \frac{1}{2}[f(\theta)]^2 d\theta$. First recognize that (although other values work, they all produce the same result) $a = -\pi/4$ and $b = 3\pi/4$. These are consecutive solutions to $\cos(\theta) + \sin(\theta) = 0$. Here is a visual:



Then the area is found by computing the integral $\int_{-\pi/4}^{3\pi/4} \frac{1}{2}[\cos(\theta) + \sin(\theta)]^2 d\theta$. By expansion, and using the facts that 1 = $\cos^2(x)+\sin^2(x)$, and also that cos(x)sin(x) = sin(2x)/2, this becomes $\int_{-\pi/4}^{3\pi/4} \frac{1}{2}(1 + \sin(2\theta))d\theta$, which equals $\pi/2$.

10. To compute more efficiently, recognize that this is the same as translating to the left by 4. Now the region is bound by y = x^2+1, x=-1, and x=2. To find the volume, evaluate the integral $\pi \int_1^2 (x^4 + 2x^2 + 1)dx$ = 78π/5.

11. This is the integral of the derivative of arcsin(x). arcsin(1)-arcsin(-1/2) = π/2 – (-π/6) = 2π/3.

12. Two pieces of information are necessary: f(g(2)) and d/dx(f(g(2))). g(2)=1, and f(1)=5, so f(g(2))=5. d/dx(f(g(2)))=g'(2)*f'(g(2)). g'(2)=-6 and f'(1)=4, so d/dx(f(g(2)))=-24. The resulting tangent line has slope of -24 and goes through the point (2,5). The result is $24x - y = 43$.

13. This is U substitution. Let u=tan(θ/2) and then du=1/2*sec^2(θ/2) dθ. ∫4udu = 2u^2 –> $2\tan^2(\theta/2)$ evaluated at 0 and π/3 results in an answer of 2/3.

14. Break it up into two integrals. ∫1/(x^2+1)dx = $\tan^{-1}(x)$+C. From u-substitution with u=$1+x^2$, we get that $\int x/(x^2+1)=1/2*\ln(x^2+1)+C$. Combine the two to get the answer of $1/2*\ln(x^2+1)+\tan^{-1}(x)+C$, which is not listed.

15. Surface area is S=π*D^2. If dS/dt = -2 = 2*π*D*dD/dt, and D=5, then dD/dt = -2/(2*π*5) = -1/(5π). Units follow accordingly, and it is asking for the rate at which the diameter decreases, so it is expressed as a positive number.

16. Every x^3y^4 comes from $(2x)^3(-y)^4(3)^2 = 72x^3y^4$. And how many of them are there? 9!/(3!4!2!). 72*9!/(3!4!2!) = 90,720.

17. It is easiest to explain in base 10. $133.3_4 = 1*4^2+3*4+3*4^0+3*4^{-1}$=16+12+3+3/4=31.75. $44.1_5 = 4*5+4*5^0+1*5^{-1}$=20+4+1/5=24.2. The difference is 7.55. 7 in base 20 is 7. .55 = 5.5/10 = 11/20, and B is the digit representing 11. Therefore the answer is $7.B_{20}$.

18. The prime factorization of 2013 is 3*11*61. Because every positive integral factor of 2013 contains either 3^0 or 3^1, and either 11^0 or 11^1, and either 61^0 or 61^1, their sum can be expressed as (3+1)*(11+1)*(61+1)=4*12*62=2976.

19. Newton's Method finds successive approximations by $x_2 = x_1 - f(x_1)/f'(x_1)$. x_1=3 is given. $f(x_1) = 7$ because the tangent line at x=3 has the same value at that point. $f'(x_1)$ equals the slope of the tangent line, which is 4. 3 - 7/4 = 5/4 = x_2.

20. The fact that in base n, A is a digit means that n ≥ 11. And the thousands digit of 15! is 0 because 15! = 5*10*15*4*[some other stuff] = 3000*[some other stuff], and therefore n < 15. 11 and 13 are primes, so they have only 1 positive integral factor, and 12 is divisible by 4. Note that 14 satisfies all other conditions.

21. On the domain specified, tanθ is defined. Notice that $x^2 = 4\frac{\sin^2\theta}{\cos^2\theta} = 4\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right) = \frac{16}{y} - 4.$ Therefore $x^2 + 4 = \frac{16}{y}$, and so $y = \frac{16}{4+x^2}$.

22. Start with 100 numbers. There are 25 primes less than 100, so we are left with 75 numbers after Brian's erasures. There are 33 multiples of 3 less than or equal to 100, but one (3) has already been erased since it is prime. After Chloe, there are therefore 75 - 32 = 43 numbers remaining. There are 20 multiples of 5 up to 100. But (5) is prime and every multiple of (15) is also a multiple of 3. So the following have already been erased: (5,15,30,45,60,75,90), leaving 13 numbers to be erased. After Denise there are 43 - 13 = 30 numbers remaining. And the multiples of 7 that are not prime, not multiples of 3 or 5, and less than 100 are: (14, 28, 49, 56, 77, 91, 98), which total 7 new erasures for Ethan. 30 - 7 = 23.

23. This is a definition of the span of a set of vectors.

24. Because of the dimensions of the matrix, k can be 1, 2, or 3. If k = 1, square matrices are obtained by deleting 2 rows and 3 columns, leaving only one entry. There are 12 entries in the original matrix. If k = 2, square matrices are obtained by deleting 1 row and 2 columns, leaving a 2x2 matrix. There are (3 choose 1) ways to delete 1 row and (4 choose 2) ways to delete 2 columns, so there are 3*6=18 minors when k=2. If k = 3, square matrices are obtained by deleting 0 rows and 1 column. This can be done in (3 choose 0)*(4 choose 1) = 4 ways. 12+18+4 = 34.

25. The easiest first step is to find two perfect squares that sum to 241. The only two are 225 and 16. This means the second and third terms are some combination of +/-4,+/-15. If both positive or both negative, this means that since they form an arithmetic sequence, the other numbers must be +/-7,-/+26, but their squares only sum to 725. This means one of (4,15) is positive and the other negative. If, for example we have __,-4,15,__, the other two numbers are -23 and 34, and their squares indeed sum to 1685. Regardless of which we choose to be positive or negative, there will be two of each in the four terms, which means the product of all four will be positive. 23*4*15*34=46920.

26. $\bar{x} = \frac{\int_a^b xf(x)dx}{\int_a^b f(x)dx} = \frac{\int_0^6 2x^{3/2}dx}{\int_0^6 2\sqrt{x}dx} = \frac{144\sqrt{6}/5}{8\sqrt{6}} = 18/5, \bar{y} = \frac{\frac{1}{2}\int_a^b (f(x))^2 dx}{\int_a^b f(x)dx} = \frac{\frac{1}{2}\int_0^6 4xdx}{\int_0^6 2\sqrt{x}dx} = \frac{36}{8\sqrt{6}} = \frac{3\sqrt{6}}{4}$.

27. For any n≥1, the sequence $\{X_1,...,X_n\}$ can be ordered in descending order. Because of the distribution of the random variables, the probability of any two numbers in the sequence being the same is 0. Since the random variables are independent and identically distributed, the probability that X_1 is the highest is the same as X_2 being the highest, which is the same as X_n being the highest. This is 1/n.

28. Consider the equivalent problem of ordering 4 black balls (numbers selected for *L*) and 6 white balls (numbers not selected for *L*). There is a "gap" between each neighboring pair of white balls and one "gap" on each end, for a total of 7 gaps. First we place 3 black balls in the gaps so that there is at most one black ball in each gap. This can be achieved in (7 choose 3) ways. The fourth black ball can then be placed in the same gap as any of the three previously placed black balls. Multiply 3 by 35 for a product of 105. This is divided by the total number of ways to arrange 6 white balls and 4 black balls (10!/6!4!) = 210. 105/210 = 1/2.

29. 9 a.m. is 0 and 5 p.m. is 8 in the new time domain. Thus the average temperature over the period of time is the 1/8*integral from 0 to 8 of 20pi/11*sin(pi*t/12)dt = 1/8*[50t - 240/11*cos(pi*t/12) | 0 to 8] = 1/8 * [400 + 360/11] = 50 + 45/11 = 54 + 1/11 = 54.09.

30. A=8, B=12, C=13, D=E=0, F=-884. Rotation angle θ: $\cot(2\theta) = \frac{A-C}{B} = -\frac{5}{12}$. Hence, using half-angle formulas, $\sin\theta = \sqrt{9/13}$, $\cos\theta = \sqrt{4/13}$. Thus, $x = \frac{2}{\sqrt{13}}\tilde{x} - \frac{3}{\sqrt{13}}\tilde{y}$ and $y = \frac{3}{\sqrt{13}}\tilde{x} + \frac{2}{\sqrt{13}}\tilde{y}$. This results in the original equation being equivalent to $17\tilde{x}^2 + 4\tilde{y}^2 = 884$. Using the equation for area of an ellipse, the area comes out to $26\pi\sqrt{17}$.