

Theta Three-Dimensional Geometry 2013 MAΘ National Convention

Note: For all questions, answer “(E) NOTA” means none of the above answers is correct.

1. What is the square of the length of the longest diagonal of a cube having edge length 3?

(A) $3\sqrt{3}$ (B) $9\sqrt{3}$ (C) 27 (D) $27\sqrt{3}$ (E) NOTA

2. A solid sphere with radius length 17 is cut by a plane so that the intersection of the sphere and the plane creates a circle with area of 225π . Find the distance between the plane and the great circle of the sphere that is parallel to the plane.

(A) 12 (B) 8 (C) 15 (D) 10 (E) NOTA

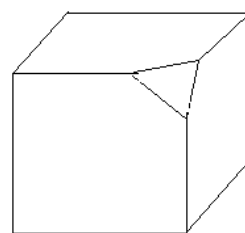
3. A tank in the shape of a right rectangular prism has a base with dimensions of 5 meters by 10 meters and a height of 14 meters. This tank is filled up half way with water. A metal cone that weighs 357 grams and has a density of 3 grams per cubic meter is dropped into the tank, causing the water level to go up. What is the positive difference, in meters, in the height of the water before and after the cone was dropped into the tank?

(A) 2.38 (B) 2.48 (C) 2.66 (D) 2.76 (E) NOTA

4. A frustum has parallel circular bases with areas 64π and 16π . The height of the frustum is 36. Find the volume of the frustum.

(A) 1776π (B) 1632π (C) 1488π (D) 1344π (E) NOTA

5. A solid cube has an edge length of 16 centimeters. A triangle is drawn with vertices at the midpoints of three connecting edges, and one “corner” of the cube is truncated (i.e., cut with a planar knife) so that this triangle is showing. The remaining figure is a solid with 7 faces. Find, in square centimeters, the surface area of this solid. (Note: Diagram to the right is not necessarily to scale.)

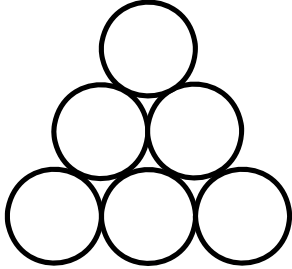


(A) $1440+32\sqrt{3}$ (B) $1440+64\sqrt{3}$ (C) $1184+32\sqrt{3}$ (D) $1440+64\sqrt{3}$ (E) NOTA

6. The cross section of a metal cylindrical pipe consists of two concentric circles with constant radius throughout length of the pipe. If the area of the annulus is 160π and the thickness of the pipe is 4, find the length of the diameter of the pipe.

(A) 11 (B) 22 (C) 44 (D) 66 (E) NOTA

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7. A *cubeohemioctahedron* is a polyhedron that has how many faces, edges, and vertices?
- (A) Number of faces = 10; Number of edges = 24; Number of vertices = 12.
(B) Number of faces = 10; Number of edges = 20; Number of vertices = 12.
(C) Number of faces = 14; Number of edges = 24; Number of vertices = 12.
(D) Number of faces = 10; Number of edges = 24; Number of vertices = 16.
(E) NOTA
8. Let A and B be opposite vertices of a unit cube (i.e., the distance between A and B is $\sqrt{3}$). Find the radius of a sphere, whose center is in the interior of the cube, that is tangent to the three faces that meet at A and also tangent to the three edges that meet at B.
- (A) $3 - \sqrt{5}$ (B) $\frac{\sqrt{3}}{3}$ (C) $2 - \sqrt{2}$ (D) $\frac{3-\sqrt{3}}{2}$ (E) NOTA
9. Joe wants to melt a solid metal sphere with radius length 12 ft. into a right circular cylinder with a base having area of 36π ft². How many feet tall will the cylinder be?
- (A) 16 (B) 24 (C) 32 (D) 64 (E) NOTA
10. Jill has congruent, perfectly spherical oranges. She arranges them in a pyramid-like structure on a level table where the base is made up of 3x3 of oranges that are tangent to each other, the second layer is made up of 2x2 of oranges that are also tangent to each other, and the top layer has 1 orange. Each orange in the top two layers is placed so that it is tangent to four oranges in the layer below. The cross-section of the resulting structure that includes the great circles of 6 oranges looks like the figure to the right. How many oranges are in the structure?
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- (A) 12 (B) 13 (C) 14 (D) 15 (E) NOTA
11. With reference to Problem #10, the maximum height of this pyramid-like structure is 4. What is the length of the radius of each individual orange?
- (A) $\frac{1}{1+\sqrt{3}}$ (B) $\frac{1}{2+\sqrt{3}}$ (C) $\frac{2}{1+\sqrt{3}}$ (D) $\frac{2}{1+2\sqrt{3}}$ (E) NOTA

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12. A cube with side length 7 is dipped in a bucket of paint. This cube is then cut into 343 unit cubes. How many cubes have exactly one side painted?
- (A) 125 (B) 135 (C) 140 (D) 150 (E) NOTA
13. What is the volume of a hexagonal pyramid with base side length of 6 and height of 3? The base of the pyramid is a regular hexagon.
- (A) $108\sqrt{3}$ (B) $54\sqrt{3}$ (C) $27\sqrt{3}$ (D) $9\sqrt{3}$ (E) NOTA
14. Find the distance between the points (17, -73, 87) and (53, 4, 2).
- (A) $5\sqrt{574}$ (B) 120 (C) $85\sqrt{2}$ (D) $95\sqrt{2}$ (E) NOTA
15. What is the total surface area of a cube with edge length equal to the radius of a sphere with volume 288π ?
- (A) 72 (B) 144 (C) 216 (D) 288 (E) NOTA
16. A sphere of radius 6 centimeters is melted into a cylindrical wire with a constant radius of 0.06 centimeters. What is the length of the wire, in meters?
- (A) 80000 (B) 8000 (C) 800 (D) 80 (E) NOTA
17. Joe wants to fit a chopstick in a cylindrical can. The radius of the can has length $m/2$, and the height of the can is equal to the slant height of a cone with radius length $m/2$ and height $2m$. If the width of the chopstick is negligible, find, in terms of m , the length of the chopstick that will exactly fit in the can.
- (A) $m\sqrt{21}/2$ (B) $m\sqrt{21}/3$ (C) $2m\sqrt{21}$ (D) $2m\sqrt{21}/3$ (E) NOTA
18. Betty cuts a circular pizza into n slices, forming n circular sectors. The measures of the central angles of these slices, when arranged from smallest to largest, form an arithmetic progression. If the smallest slice has an angle measure of 3° and the largest has an angle measure of 37° , how many slices did Betty cut?
- (A) 14 (B) 15 (C) 16 (D) 17 (E) NOTA
19. A right circular cone is cut by a plane parallel to the base, splitting the cone into a smaller cone and a frustum. The frustum has bases with diameter lengths of 6 and 30. The height of the frustum is 12. What is the slant height of the smaller cone?
- (A) 3 (B) $3\sqrt{2}$ (C) $5/2$ (D) $5\sqrt{2}/2$ (E) NOTA

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20. Ignoring units, find the sum of the lateral surface area and the volume of the frustum in Problem #19.
- (A) $\pi(1116+ 288\sqrt{2})$ (B) $\pi(1116+ 216\sqrt{2})$
- (C) $\pi(837+ 288\sqrt{2})$ (D) $\pi(837+ 216\sqrt{2})$ (E) NOTA
21. The ratio between the radii of two spheres is 5:3. What is the ratio of the volume of the larger sphere to the volume of the smaller sphere?
- (A) $25\sqrt{2}/3$ (B) $25/9$ (C) $125\sqrt{2}/9$ (D) $125/27$ (E) NOTA
22. Two spherical balls lie on the ground tangent to each other. The radius of one ball is 18 feet and the radius of the other is 24 feet. What is the height, in feet, from the ground to the point of tangency of the two spheres?
- (A) $108/7$ (B) $144/7$ (C) $216/7$ (D) $288/7$ (E) NOTA
23. What is the total surface area of a pyramid with a square base where the side of the base is 10 and the height of the pyramid is 12?
- (A) 300 (B) 330 (C) 345 (D) 360 (E) NOTA
24. A light bulb is in the shape of a cube. It has an edge length of six centimeters and can only illuminate a maximum distance of three centimeters from any point of the cube. What is the maximum total space, in cubic centimeters, that the light bulb can light up that does not include the inside of the cube?
- (A) $648+198\pi$ (B) $648+214\pi$ (C) $648+368\pi$ (D) $648+228\pi$ (E) NOTA
25. What is the slant height of a right circular cone whose base has a circumference of 170π and height of 132?
- (A) 157 (B) $2\sqrt{6117}$ (C) $11\sqrt{203}$ (D) $9\sqrt{327}$ (E) NOTA
26. What is the volume of a regular tetrahedron with side length 4?
- (A) $4\sqrt{2}/3$ (B) $8\sqrt{2}/3$ (C) $4\sqrt{3}/3$ (D) $8\sqrt{3}/3$ (E) NOTA
27. What is the surface area of a regular icosahedron with side length 12?
- (A) $540\sqrt{3}$ (B) $720\sqrt{3}$ (C) $810\sqrt{3}$ (D) $1080\sqrt{3}$ (E) NOTA

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28. Kelly wants to paint the walls and the roof of her room, which is in the shape of a right rectangular prism. Her room is 18 feet by 21 feet, and is 12 feet tall. What is the surface area she needs to cover, in *square yards*?

- (A) 146 (B) 164 (C) 156 (D) 165 (E) NOTA

29. A sphere is inscribed in a tetrahedron with vertices at $(4,0,0)$, $(0,3,0)$, $(0,0,1)$, and $(0,0,0)$. The volume of the sphere can be expressed in the form $m\pi/n$, where m and n are relatively prime positive integers. Find $m + n$.

- (A) 141 (B) 139 (C) 137 (D) 135 (E) NOTA

30. A sealed container is in the shape of a right circular cone that is 4 inches tall and the base of the cone has a radius of 3 inches. There is water in the container, and when the container is held with its vertex down and base horizontal, the water level is 2 inches away from the vertex. When the container is held with its vertex up and base horizontal, the water level is D inches away from the vertex. Find the value of D .

- (A) $2 \cdot \sqrt[3]{3}$ (B) $2 \cdot \sqrt[3]{5}$ (C) 2 (D) $2 \cdot \sqrt[3]{7}$ (E) NOTA