1. Let A, B, C, D, E be the 5 integers.

$$\frac{\text{sum}}{\text{average}} = \frac{A+B+C+D+E}{\left(\frac{A+B+C+D+E}{5}\right)} = \frac{x}{100}$$

$$100(A+B+C+D+E) = x \cdot \frac{A+B+C+D+E}{5}$$

$$100 = \frac{x}{5}$$

$$x = 500$$

2. $x = 1+3i$ or $x = 1-3i$

$x-1-3i = 0$ or $x-1+3i = 0$

$$((x-1)-3i)((x-1)+3i) = 0$$
$$(x-1)^2 - (3i)^2 = 0$$
$$x^2 - 2x + 1 + 9 = 0$$
$$x^2 - 2x + 10 = 0$$
$$1 + (-2) + 10 = 9$$

3. Let x = width of the frame. $\frac{36-2x}{48-2x} = \frac{5}{7}$

$$7(36-2x) = 5(48-2x)$$
$$x = 3$$

4. $\left(\frac{1}{1/y + 1/x}\right)(x+y) = \left(\frac{1}{\frac{x+y}{xy}}\right)(x+y) = \left(\frac{xy}{x+y}\right)(x+y) = xy$

5. If $f^{-1}(3) = -2$ then $f(-2) = 3$.

$$f(-2) = k\left(2-(-2)-(-2)^3\right) = 3$$
$$= k(12) = 3$$
$$k = \frac{1}{4}$$

6. If $y=\dfrac{x-2}{x-1}$ switch the x and the y's and solve for x.

$$\begin{aligned} x &= \frac{y-2}{y-1} \\ x(y-1) &= y-2 \\ xy-x &= y-2 \\ xy-y &= x-2 \\ y(x-1) &= x-2 \\ y &= \frac{x-2}{x-1} \end{aligned}$$

7. $f(t)=-4.9t^2+10t+2$ The maximum height occurs at $t=\dfrac{-b}{2a}=-\dfrac{10}{2(-4.9)}=\dfrac{50}{49}$. The maximum height is

$$f\left(\frac{50}{49}\right)=-4.9\left(\frac{50}{49}\right)^2+10\left(\frac{50}{49}\right)+2=\frac{-49}{10}\cdot\frac{50}{49}\cdot\frac{50}{49}+10\cdot\frac{50}{49}+2=-5\cdot\frac{50}{49}+\frac{500}{49}+\frac{98}{49}=\frac{-250+500+98}{49}=\frac{348}{49}$$

$$49\overline{)348}\quad 7.11$$

8. $\dfrac{3}{x+1}+\dfrac{2}{x}=\dfrac{3x+2(x+1)}{x(x+1)}=\dfrac{5x+2}{x^2+x}$ so the reciprocal is $\dfrac{x^2+x}{5x+2}$.

9. $(x+y)^7=x^7+7x^6y+21x^5y^2+35x^4y^3+21x^2y^5+7xy^6+y^7$

$(x-y)^7=x^7-7x^6y+21x^5y^2-35x^4y^3+21x^2y^5-7xy^6+y^7$

$(x+y)^7+(x-y)^7=2x^7+42x^5y^2+42x^2y^5+2y^7$

10. $\dfrac{x+4}{(x+1)(x+2)}=\dfrac{Q}{x+1}+\dfrac{R}{x+2}$

$$\begin{aligned} x+4 &= Q(x+2)+R(x+1) \\ x+4 &= Qx+2Q+Rx+R \\ x+4 &= (Q+R)x+(2Q+R) \end{aligned}$$

$$\begin{cases} Q+R=1 \\ 2Q+R=4 \end{cases}$$

11. By completing the square, we find the equation of the circle in standard form is $(x+5)^2+(y-3)^2=18$. The center of the circle is (-5,3). The slope of the radius from the center of the circle to the point of tangency is $m=\frac{6-3}{-2+5}=\frac{3}{3}=1$, so the slope of the line that is tangent to the circle at (-2, 6) is $m=-1$. Therefore, $y-6=-1(x+2)$ changed to slope-intercept form is $y=-x+4$.

12. $\frac{n(n+1)!-2(n!)}{(n+1)!+n!}=\frac{n(n+1)n!-2n!}{(n+1)n!+n!}=\frac{n!(n(n+1)-2)}{n!(n+1+1)}=\frac{n^2+n-2}{n+2}=\frac{(n+2)(n-1)}{(n+2)}=n-1$

13. $f(x)=\sqrt{9-x^2}$ represents a semicircle with center at (0,0) and a radius of 3. Possible x-values (domain) range from -3 to 3 inclusive, and the possible y-values (range) range from 0 to 3 inclusive.

14. I. Definition of an even function.
II. Definition of an odd function.
III. Obvious
IV. y = 0 is both even and odd.
V. $f(x)=\frac{1}{2}\left(f(x)+f(-x)\right)+\frac{1}{2}\left(f(x)-f(-x)\right)$
$g(x)=f(x)+f(-x)$ is an even function; $h(x)=f(x)-f(-x)$ is an odd function.

15. $f(x)=ax^4+x^3-cx^2+5x+1$

$$f(4)=256a+64-16c+20+1$$
$$f(-4)=256a-64-16c-20+1$$
$$f(4)-f(-4)=128+40$$
$$f(4)-10=168$$
$$f(4)=178$$

16. A: $g = \frac{kM^2}{d^2}$ B: $g = \frac{kM^2}{2d^2}$ C: $g = \frac{k2M^2}{d^2}$ D: $g = \frac{k4M^2}{d^2}$

D has the largest numerator with the smallest denominator.

17. $f(x) = \frac{1}{1-x}$ $f(f(x)) = \frac{1}{1-\frac{1}{1-x}} = \frac{x-1}{x}$ $f(f(f(x))) = f\left(\frac{x-1}{x}\right) = \frac{1}{1-\frac{x-1}{x}} = x$

$$g(x) = \begin{cases} x, x \neq 0,1 \\ \text{undefined}, x = 0,1 \end{cases}$$

18. $f(x) = |x-3| + |x-1|$

$$\begin{aligned}
&f(t+2) = f(t) \\
&|t+2-3| + |t+2-1| = |t-3| + |t-1| \\
&|t-1| + |t+1| = |t-3| + |t-1| \\
&|t+1| = |t-3| \\
&t+1 = t-3 \Rightarrow \text{never} \\
&t+1 = -(t-3) \Rightarrow t = 1
\end{aligned}$$

19. LHS: $x^2y \to (1.25x)^2(0.8y) = 1.25x^2y$; RHS: $\frac{z}{4y^2} \to \frac{z}{4(0.8y)^2} = \frac{az}{4(0.64y^2)}$.

$1.25x^2y = a\frac{z}{4(0.64y^2)} \to a = 0.8$ so z is decreased by 20%

20. Draw the semicircle with the center at the origin and the flat part of the semicircle on the x-axis. The line $y = \frac{R}{2}$ is the line that is drawn across the stage half-way from the front to the back. $y = \frac{R}{2}$ intersects the semicircle at two points forming a chord parallel to and at a height of $\frac{R}{2}$ above the x-axis. The radii from the origin to the ends of this chord form an isosceles triangle which can be vertically bisected into two right triangles. The height of each triangle is $\frac{R}{2}$ and the hypotenuse is R. Therefore, the lowermost angle of each right triangle is 60°. The vertex angle of the isosceles triangle is 120° and the area of the isosceles triangle is $\frac{1}{2}\cdot\left(R\sqrt{3}\cdot\frac{R}{2}\right) = \frac{R^2\sqrt{3}}{4}$. The radii also define a sector of the semicircle with area of 2/3 times the area of the stage: $\frac{2}{3}\cdot\frac{\pi}{2}\cdot R^2 = \frac{\pi}{3}\cdot R^2$. The area of the front section of the stage is a segment of the semicircle with area $\frac{\pi}{3}\cdot R^2 - \frac{R^2\sqrt{3}}{4} = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)R^2$. The fraction of the stage's area in front of the line is $\frac{\frac{\left(4\pi - 3\sqrt{3}\right)}{12}\cdot R^2}{\frac{\pi R^2}{2}} = \frac{4\pi - 3\sqrt{3}}{6\pi}$.

21. Substitute each (x, y) into $y = ax^2 + bx + c$ to form the following system of equations:

$$9a + 3b + c = 12$$
$$4a - 2b + c = -3$$
$$16a + 4b + c = 21$$

By Gaussian Elimination, $a = 1$, $b = 2$, $c = -3$, so the equation of the parabola is $y = x^2 + 2x - 3$. When $x = 2$, $y = 5$.

22. Let n = the number of \$5.00 price decreases. Profit = revenue-cost .
profit=(Price of cameras)*(number of cameras)-(cost for camera)*(number of cameras)
profit = $(120-5n)(21+3n)-75(21+3n)=945+30n-15n^2$

23. $g\circ h=\{(3,2),(1,-1),(-3,3)\}$ so $f\circ g\circ h=\{(3,5),(1,1)\}$

24. At the beginning, the mixture is composed of 9 gallons of water, 2 gallons of oil, and 19 gallons of ethanol. Since the amount of water remains constant, we can write $.2x=.3(30)$ where x = the volume of the new mixture. This means x = 45 gallons. The new mixture will contain 9 gallons of water, 7 gallons of oil and 29 gallons of ethanol. Since the mixture already has 19 gallons of ethanol, she needs to add 10 gallons.

25. The length of the major axis (2a) is 10, so a = 5. The distance from the center to the foci is c = 4. In an ellipse, $a^2-b^2=c^2\Rightarrow 25-b^2=16\Rightarrow b=3$. The length of the minor axis is $2b$=6

26. Let $x=\sqrt[4]{a}$. We can rewrite the equation as $2x^2-13x+20=0$. Since $x=\frac{5}{2}$ or 4,
$a=\frac{625}{16}$ or 256. $\frac{625}{16}+256=\frac{4721}{16}$

27. The matrix describes a circle with radius = 5 and a hyperbola with the x- and y- axis as its asymptotes. The circle and hyperbola will intersect 4 times (twice in the first quadrant and twice in the third). The intersection points are (2,1), (1,2), (-2,-1), (-1 -2).

28. There are 3 ways these four digits add to 7:
1. One 4 and three 1's -> 4 permutations
2. Three 2's and one 1 -> 4 permutations
3. One 3, one 2, and two 1's -> 12 permutations

29. Since distance = rate x time, time = distance/rate. Let x = my speed. My time (in hours) is $\frac{100\text{km}}{x\,\text{km/hour}}$. My sister's time is $\frac{100\text{km}}{(x-10)\,\text{km/hour}}$. Therefore, $\frac{100}{x}=\frac{100}{x-10}-\frac{1}{2}$.

i.e. my time = my sister's time – half an hour. Solving the resulting quadratic gives us 50 km/hour.

30. Let $x=\frac{1}{2}$ | Let $x=1$

$5f(2)+4f(1)=\frac{1}{2}$ | $5f(1)+f(2)=1$

If we solve this system of equations, $f(1)=\frac{3}{14}$

Bonus: $x^2-4x+12=y$. $12-y=4$ -> $y=8$